
ORBIS

Modern Rolling-Element Bearing Analysis Software

USER'S MANUAL

Updated for Version 3.0
Jul-2018

Table of Contents

1	Getting Started.....	5
1.1	System Requirements	5
1.2	Installation Instructions	5
1.3	Java Runtime Environment	5
2	General Data Input and Conventions	5
2.1	Coordinate Systems	5
2.2	Numerical Input Formatting	6
2.3	Input Field Expressions/Calculations	6
2.4	Unit Conversions.....	7
3	User Interfaces	8
3.1	Main Graphical User Interface (GUI)	8
3.1.1	System Inputs	10
3.1.2	Dynamic Analysis Inputs	10
3.1.3	System Display.....	11
3.1.4	Bearing Row Inputs.....	12
3.1.4.1	Bearing Selection Dialog	14
3.1.4.2	Contact Angle Orientation	14
3.1.5	Dashboard	15
3.1.5.1	Console Window.....	16
3.1.6	User Menus	16
3.1.6.1	Batch Process Load Cases	17
3.1.6.2	Shaft Flexibility	18
3.1.6.3	Sensitivity Studies.....	20
3.1.6.3.1	Sensitivity Plot Windows	22
3.1.6.4	Tolerance Studies.....	22
3.1.6.5	Dahl Torque Hysteresis.....	23
3.1.6.6	Enforced Shaft Deflections	25
3.1.6.7	System Preferences	25
3.1.6.8	Load License File.....	27
3.1.6.9	About	27
3.2	Database Editors.....	27
3.2.1	Bearing Database Editor.....	28
3.2.1.1	Angular Contact and Radial Bearing Terminology	31
3.2.1.2	Shoulder Height Definitions	31
3.2.1.3	4PT Gothic Arch Bearings.....	32
3.2.2	Material Database Editor.....	34
3.2.3	Lubricant Database Editor.....	35
3.3	Analysis Results Window.....	37
4	Brief Technical Background.....	38
4.1	Preloading.....	39
4.1.1	Preload Condition.....	39
4.1.2	Preload Type	40
4.2	Convergence Criteria.....	40
5	Output Descriptions.....	41
5.1	Results Summary.....	41
5.2	Input Parameters	41
5.3	Material Properties	41
5.4	External Applied Loads.....	41
5.5	Ball Crossing Angles.....	41

5.6	Internal Clearances	42
5.7	Bearing Ring Properties	42
5.8	Preload Data	44
5.9	Reaction Forces on Shaft.....	45
5.10	Inner Ring Displacements.....	45
5.11	Stiffness Outputs.....	45
5.11.1	Axial Stiffness with Ring Compliance.....	45
5.11.2	System Jacobian	45
5.11.3	Stiffness at Load Point	46
5.11.4	Row Stiffness Matrix	46
5.12	Fatigue Life.....	46
5.13	Bearing Torque	46
5.14	Ball Excursions	47
5.15	Row Outputs (Element-Wise results)	47
5.15.1	Element Number	47
5.15.2	Normal Ball Load.....	47
5.15.3	Contact Angle.....	47
5.15.4	Mean Hertz Stress	47
5.15.5	Truncation Analysis	47
5.15.6	Truncated Length	48
5.15.7	Ellipse Semi Major.....	48
5.15.8	Ellipse Semi Minor	48
5.15.9	Max Sub-Surface Shear.....	48
5.15.10	Max Shear Depth.....	48
5.15.11	Upper Edge Location.....	48
5.15.12	Lower Edge Location	48
5.15.13	Contact Normal Approach.....	48
5.15.14	Contact Normal Stiffness.....	48
5.15.15	Spinning Velocity	48
5.15.16	Rolling Velocity	49
5.15.17	Spinning Torque	49
5.15.18	Rolling Torque.....	49
5.15.19	Element Roll Velocity	49
5.15.20	Pitch Orbit Velocity.....	49
5.15.21	Minimum Film Height.....	49
5.15.22	Minimum Lambda Value.....	49
5.15.23	Centrifugal Force	49
5.15.24	Gyroscopic Moment	49
5.15.25	Ball Spin Axis Angle.....	50
6	References.....	51

Table of Figures

Figure 2.1-1.	Coordinate System	6
Figure 3.1-1.	Main Graphical Interface	8
Figure 3.1-2.	Main Window Input Categories	9
Figure 3.1-3.	System Inputs	10
Figure 3.1-4.	Dynamic Analysis Inputs	11
Figure 3.1-5.	System Display.....	12
Figure 3.1-6.	Bearing Row Inputs.....	13

Figure 3.1-7. Bearing Selection Dialog	14
Figure 3.1-8. Convergent and Divergent Contact Angles.....	15
Figure 3.1-9. Dashboard Area.....	15
Figure 3.1-10. Console Window	16
Figure 3.1-11. User Menu's.....	17
Figure 3.1-12. Batch Processing Load Cases.....	18
Figure 3.1-13. Flexible Shaft Window.....	19
Figure 3.1-14. Sensitivity Studies Dialog	21
Figure 3.1-15. Plot Windows	22
Figure 3.1-16. Tolerance Studies Dialog	23
Figure 3.1-17. Dahl Torque Hysteresis Utility	24
Figure 3.1-18. Enforced Displacements.....	25
Figure 3.1-19. System Preferences Dialog.....	26
Figure 3.1-20. Load License File Dialog	27
Figure 3.2-1. Database Editors.....	28
Figure 3.2-2. Bearing Database Inputs, Angular Contact & Radial Bearings	29
Figure 3.2-3. Bearing Database, 4PT Gothic Arch Bearings.....	30
Figure 3.2-4. Angular Contact (2PT) Terminology	31
Figure 3.2-5. Shoulder Height Definitions for h/d Values.....	32
Figure 3.2-6. Gothic Arch (4PT) Arching/Shim Terminology	33
Figure 3.2-7. 4PT Radial Play, Resting Angle and Free Contact Angle Terminology.....	34
Figure 3.2-8. Material Database Inputs.....	35
Figure 3.2-9. Lubricant Database Inputs.....	36
Figure 3.3-1. Results Window	38
Figure 5.7-1. Bearing Nomenclature for Equivalent Ring Calculations	43

Table of Tables

Table 2.2-1. Numeric Input Formatting Examples	6
Table 2.3-1. Input Field Math Operators	7
Table 2.4-1. Unit Conversion Syntax.....	7
Table 4.2-1. Solver Convergence Criteria	41
Table 5.6-1. Internal Clearance Output Descriptions.....	42
Table 5.7-1. Bearing Ring Output Descriptions	44
Table 5.8-1. Preload Output Descriptions.....	45

1 Getting Started

1.1 System Requirements

The following minimum system requirements are needed to run ORBIS.

- Windows family operating system
- Display monitor with minimum resolution of 1024 by 768 pixels
- 128 MB of free disk space
- 256 MB RAM
- Available USB port
- Java Runtime Environment 7.0 or greater (see below for more details)

1.2 Installation Instructions

Run the automated installer steps below to complete installation. Administrator rights are needed to complete the installation properly. Note: ORBIS will install to 'All Users' on a given machine.

1. Obtain installation files from developer
2. Double click the *Setup.exe*
3. Follow installer instructions to complete installation. Note: user has option to define where the application is installed. The default location is C:\Program Files (x86)\Orbis.
4. Copy license file (license.dat) provided by developer to the root installation directory.

1.3 Java Runtime Environment

In order for the software to run properly, the host computer must have Java Runtime Environment (JRE) version 7.0 or greater installed. The JRE is an industry standard and will generally already be installed on most modern computers. If your computer does not have JRE already installed you can download the latest version from the Sun/Oracle website (www.oracle.com).

2 General Data Input and Conventions

Proper bearing analysis within ORBIS requires various data inputs and conventions to be followed. This section discusses such matter and it is highly recommended that the first-time user read through this section carefully.

2.1 Coordinate Systems

ORBIS uses a standard right-handed coordinate system for both system setup and analysis results. The user will define the location of the system origin but cannot change its orientation. As shown in the figure below, the x-axis is always aligned with the shaft spin axis, with positive direction pointing rightward on the page, and the positive y-axis is defined as pointing upward on the page. Positive moments/rotations follow right hand rule along respective axes.

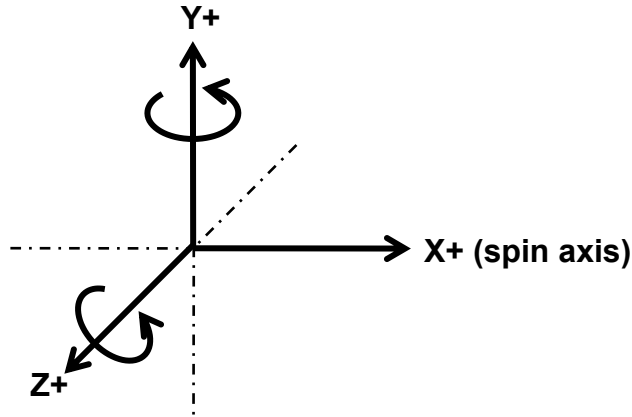


Figure 2.1-1. Coordinate System

2.2 Numerical Input Formatting

ORBIS accepts multiple different methods of numerical inputs but there are a few that are not allowed. In general, the U.S. convention (i.e. decimal point) is allowed while the European convention (comma) is not. The table below shows examples of valid and invalid numeric input formatting.

Table 2.2-1. Numeric Input Formatting Examples

Number to input	Valid Inputs	Invalid Inputs	Description
One thousand	1000, +1000, 1e3, 1.00e3, 1.0e+3, 1.0E3	1,000, 1.0e 3, 1.0x10^3	<ul style="list-style-type: none"> Plus signs are optional for positive values Scientific notation uses the "e" or "E" to specify powers of 10 Comma notation is not allowed Spaces anywhere in input string are not allowed
One thousandth	0.001, .001, 1.0e-3, 1E-3		<ul style="list-style-type: none"> Leading and trailing zeros are optional

2.3 Input Field Expressions/Calculations

Most input fields provide a built-in calculator that can interpret basic math functions. To use this functionality the user simply types the desired equation into the input field, hits the <Enter> key, and ORBIS automatically returns the answer back to the same input field. Note: the 'Enter' key must be used to tell ORBIS to parse the input field for expressions.

As an example, suppose you are defining a new bearing and want the pitch diameter to be halfway between the bearing I.D. (1.375 dia.) and O.D. (1.875 dia.). One way to accomplish this is to type $(1.375+1.875)/2$ in the pitch diameter field and hit the <Enter> key. Orbis will return the result of 1.625 to the field.

Allowable mathematical operators are shown in the table below.

Table 2.3-1. Input Field Math Operators

Operator	Mathematical Function
+	Addition
-	Subtraction
*	Multiplication
/	Division
()	Parentheses are used to group values and control order of operations
^	Power or exponent

2.4 Unit Conversions

Most ORBIS input fields contain a built-in unit converter that allows the user to define inputs in various units and have them automatically converted to the proper system units. To perform a unit conversion the user must append the unit abbreviation within square brackets to their input value. Note: this functionality may also be used in conjunction with an input expression.

The following example illustrates a conversion of 100 Newtons to the default ORBIS load units (lbf). Note that a space between the number and the square bracket is optional.

100 [N] <Enter>

Available unit abbreviations are shown in the table below. In general, both uppercase and lowercase options exist for all available units.

Table 2.4-1. Unit Conversion Syntax

Unit Type	Abbreviation	Unit	ORBIS Default Unit
Force	[N], [n]	Newton force	Pound Force
	[GF], [gf]	Gram force	
	[OZF], [ozf]	Ounce force	
Length	[MM], [mm]	Millimeter	Inch
	[CM], [cm]	Centimeter	
	[M], [m]	Meter	
	[FT], [ft]	Foot	
Pressure	[PA], [pa]	Pascal	Pound Per Square Inch
	[MPA], [mpa]	Megapascal	
	[GPA], [gpa]	Gigapascal	
Torque	[NM], [nm]	Newton*meter	Inch Pound Force
	[NCM], [ncm]	Newton*centimeter	
	[NMM], [nmm]	Newton*millimeter	
	[FTLBF],[ftlbf]	Foot*pound-force	
	[INOZF], [inozf]	Inch*ounce-force	
	[GFCM], [gfcmm]	Gram-force*millimeter	
	[GFMM], [gfmm]	Gram-force*millimeter	
Angle	[RAD], [rad]	Radian	Degrees
Rotational Velocity	[RPS], [rps]	Radians per second	Rotation Per Minute
	[DEGPS], [degps]	Degrees per second	

3 User Interfaces

This section describes each of the graphical interfaces within ORBIS.

3.1 Main Graphical User Interface (GUI)

The main graphical interface (see Figure 3.1-1) is used to configure a system of bearings, apply external loading, and submit the setup for analysis. As depicted in Figure 3.1-2, the main window is organized to group key input types for easy access. Each of these regions are discussed in subsequent sections.

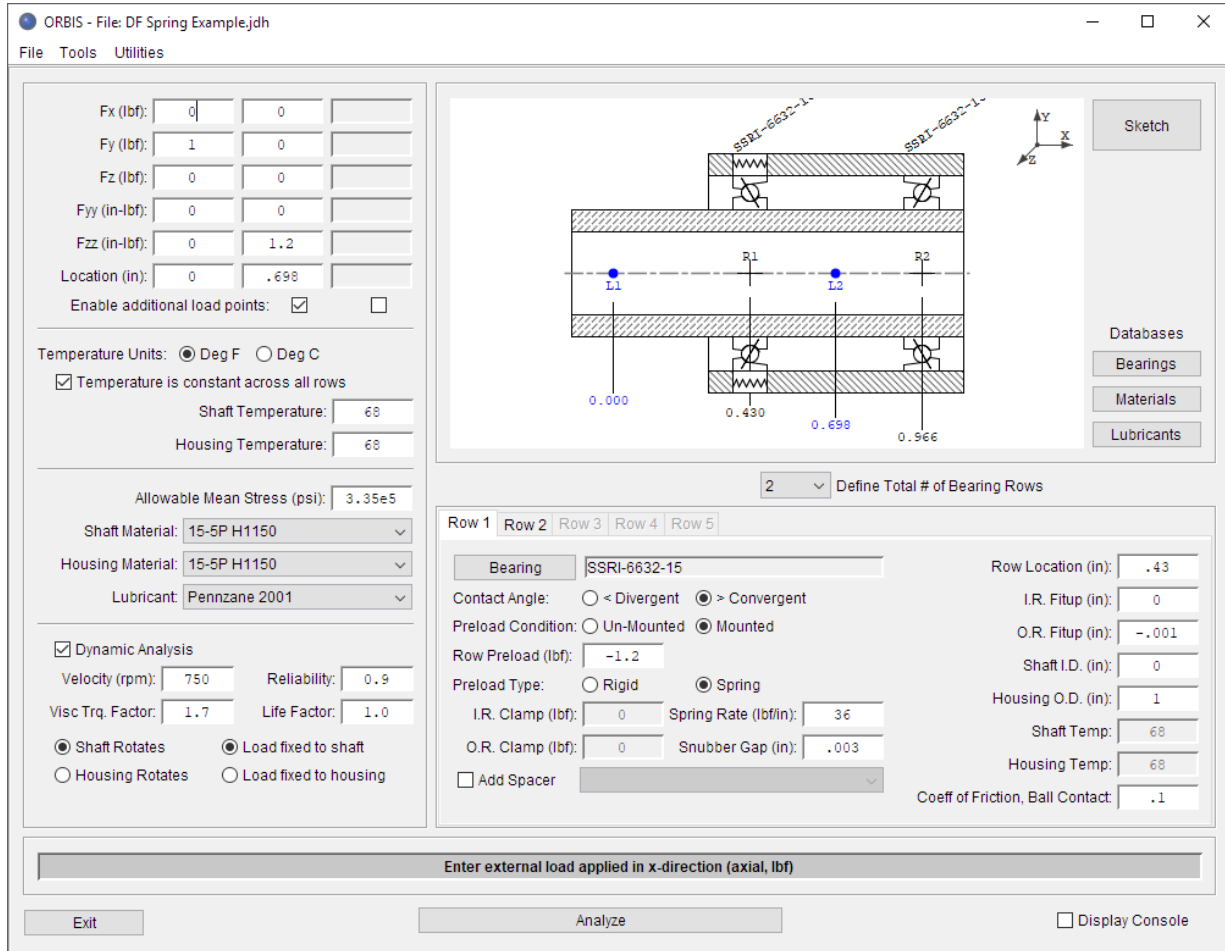


Figure 3.1-1. Main Graphical Interface

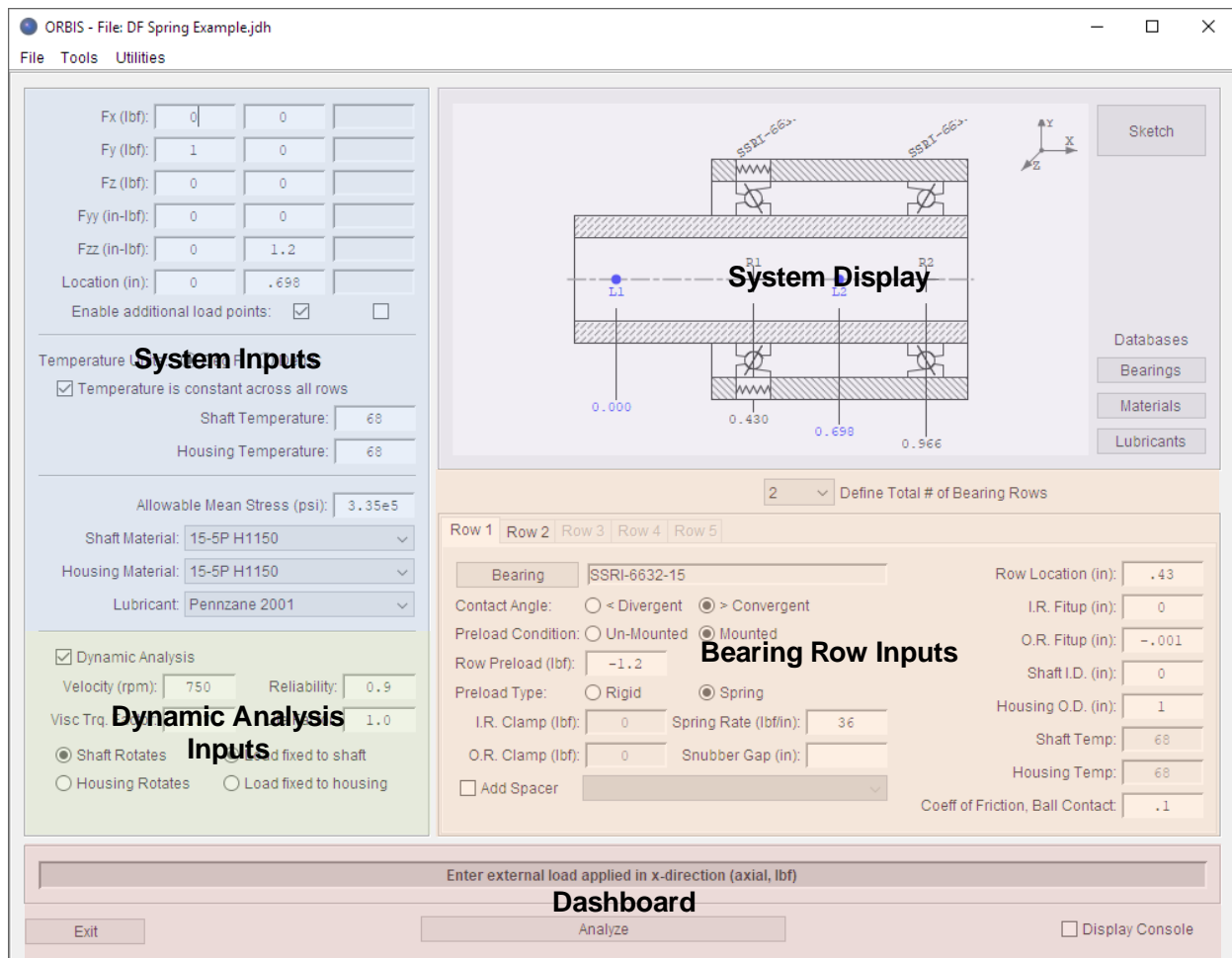


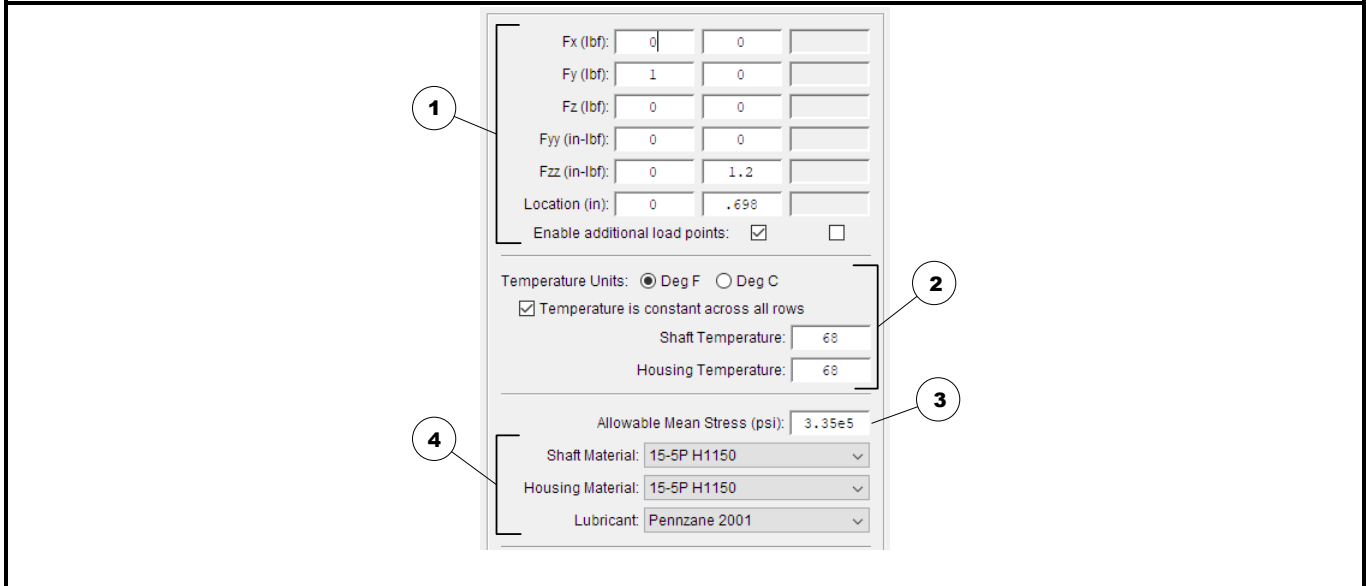
Figure 3.1-2. Main Window Input Categories

As shown in Figure 3.1-2, the main window contains the following regions:

- **System Inputs** – located in the upper left region, is where external loading, temperatures, shaft/housing materials and lubricants are defined. Refer to section 3.1.1 for details.
- **Dynamic Analysis** – located in the lower left, is where parameters such as velocities, fatigue life (reliability and life factor), viscous torque factor, rotational member and load fixity are defined. Refer to section 3.1.2 for details.
- **System Display** – located in the upper right, provides an engineering sketch based on the user defined system. Refer to section 3.1.3 for details.
- **Bearing Row Inputs** – located in the lower right, is where parameters for defining the configuration of each bearing row in the system. Refer to section 3.1.4 for details.
- **Dashboard** – located bottom center, provides key details and helpful information for each input field and contains the ‘Analyze’ button for job submittal. Refer to section 3.1.5 for details.

3.1.1 System Inputs

The System Inputs region contains inputs for external loading, bulk temperature definitions, and assignment of housing/shaft/lubricant materials. See the following figure for descriptions of each input field.



#	Title	Description
1	External Loads	External loading has the following features: <ul style="list-style-type: none"> • Full 5 dof load component specification (Fx, Fy, Fz, Fyy, and Fzz follow system coordinate definition) • Load location is the x-axis coordinate of the load • Up to 3 independent loads can be defined by selecting the check boxes.
2	Temperature Assignment	Radio button toggles between Fahrenheit and Celsius units. Checkbox specifies if temperature should be constant across all rows. If not, temperature values are assigned separately for each bearing row (see section 3.1.4).
3	Allowable Mean Hertzian Stress (psi)	Allows user to specify an allowable contact stress. All elements with contact stress above the specified allowable will be highlighted in the output file.
4	System Materials	System materials consist of the shaft, housing and lubricant materials. Each of these are assigned via drop-down selections. Available selections are based on user database definitions (see section 3.2 for discussion on databases).

Figure 3.1-3. System Inputs

3.1.2 Dynamic Analysis Inputs

ORBIS provides both static and dynamic analysis modes. Static mode is useful for simple slow speed applications where dynamic effects are negligible. The static solver is the most robust and quickest due to the reduction in equilibrium equations. Dynamic analysis mode provides full analysis output parameters (such as torque, fatigue life, film parameters, centrifugal and gyroscopic forces, etcetera). See the following figure for descriptions of the inputs and controls in this region.

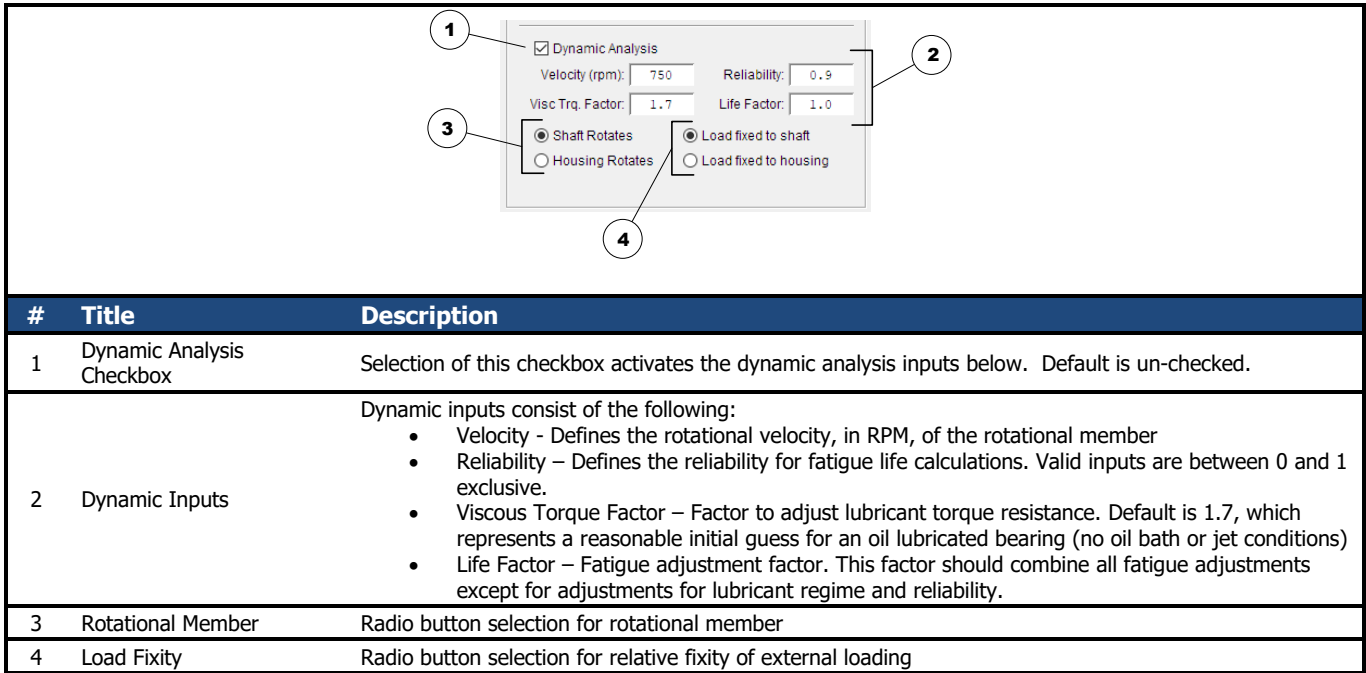


Figure 3.1-4. Dynamic Analysis Inputs

3.1.3 System Display

The system display area provides a sketch of the user defined system. This sketch takes key user input entries, such as bearing locations, load locations, bearing contact angle orientations, and generates an engineering layout sketch of the user’s setup. Review of the sketch prior to analysis submittal is advised as this provides a quick and easy means to check that the system of bearings is configured as intended by the user. Furthermore, the system sketch is copied and included in the results window as a figure to provide a complete analysis report. See the following figure for a detailed description of the information provided in the system display panel.

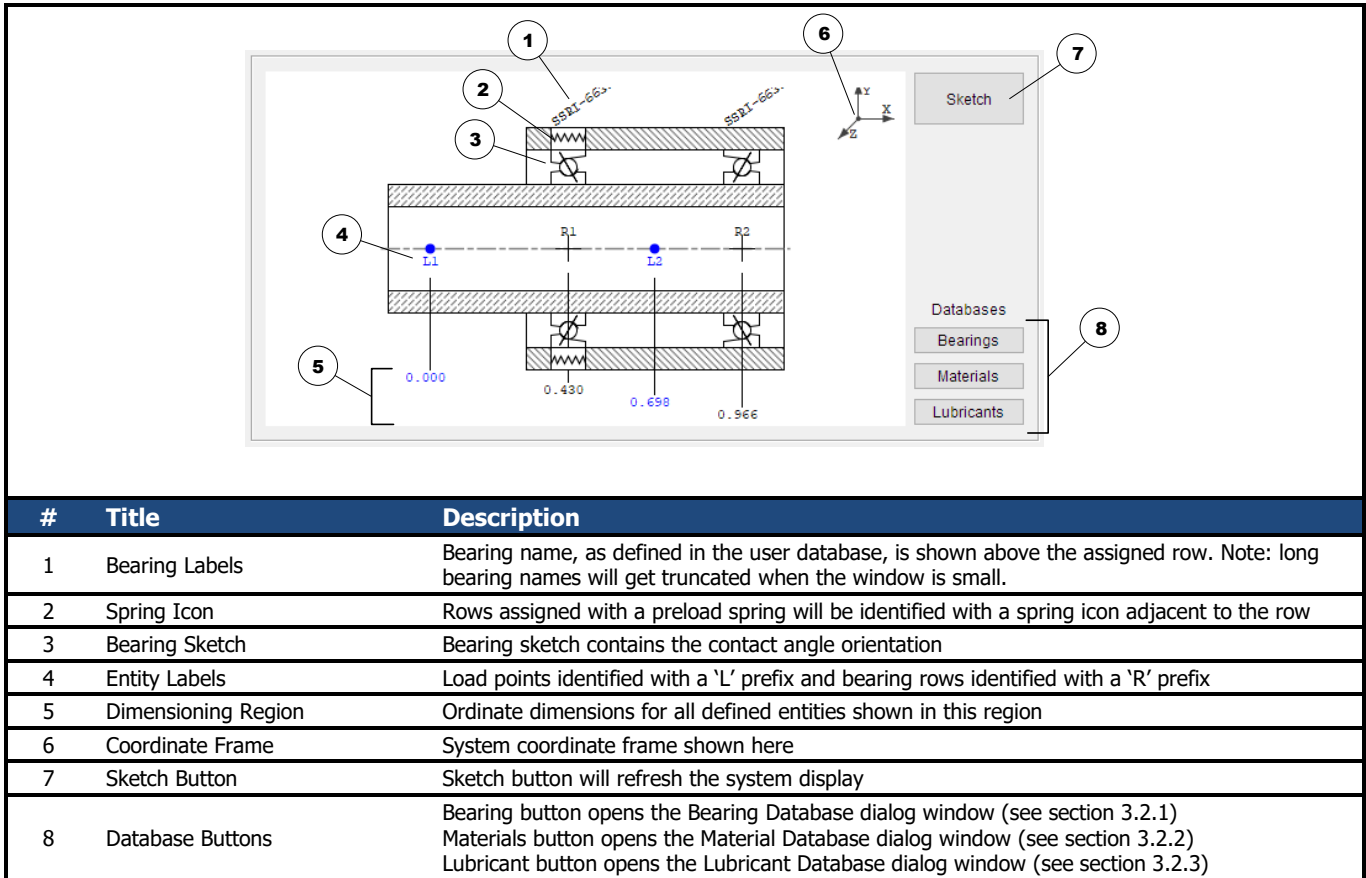


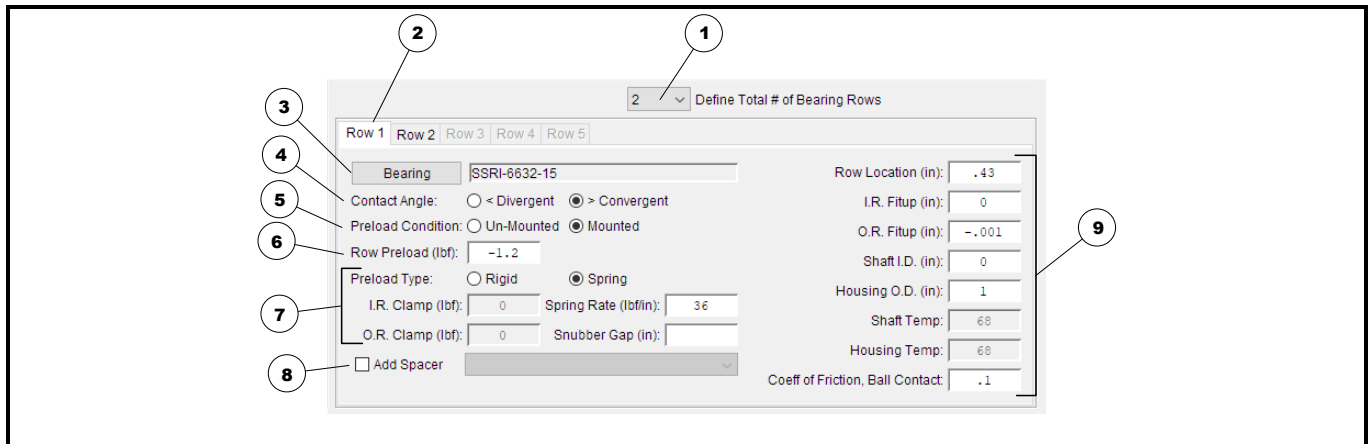
Figure 3.1-5. System Display

3.1.4 Bearing Row Inputs

The Bearing Row Input area consists of tabbed window panels used to define inputs for each bearing row in the system. See the following figure for a detailed description of each input type.

A common mistake for new users is improper sign convention on the 'Row Preload' field. The user must consider the orientation of the contact angle and specify an appropriate sign on the row preload input. Contact angles are defined using convergent and divergent terminology (see section 3.1.4.2 for details). These terms relate to whether the contact angle line of action converges or diverges toward the spin axis as you traverse along the positive x-axis (rightward along the spin axis). For example, the leftmost bearing in a duplex pair of bearings, configured in a back-to-back (DB) orientation, would have a divergent contact angle (there would be a residual force acting on the inner ring toward the right or X+ in ORBIS coordinates). Subsequently, the rightmost bearing in this hypothetical DB pair has a convergent contact angle and requires a preload force acting to the left, or negative in ORBIS coordinates, on the inner ring.

Another criterion for preloading is that the sum of all row preload values should be zero. If this is not the case there will be a net axial force in the system that gets carried into the system equilibrium solver. Such a scenario will result in bearing shifting to try and find equilibrium.

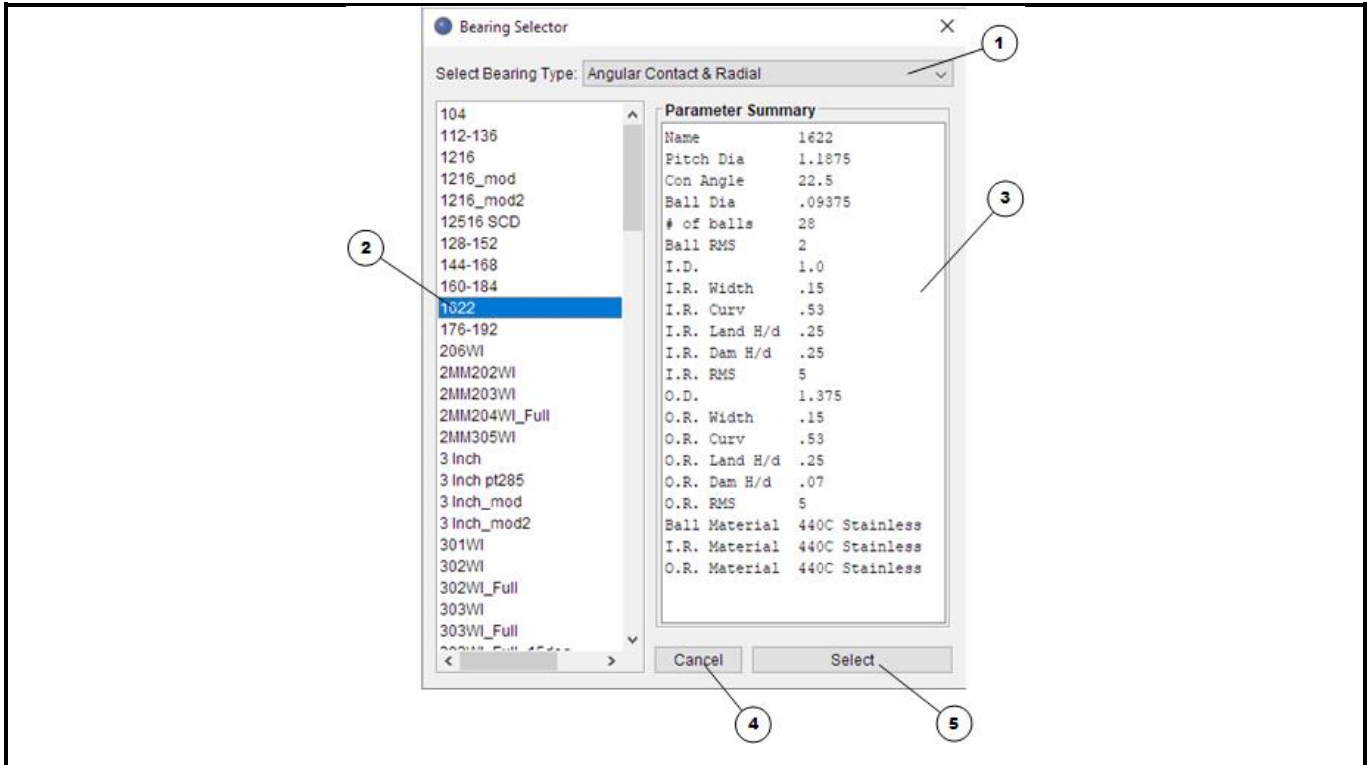


#	Title	Description
1	No. Bearing Rows	Drop-down selection allows up to 5 bearing rows on a common shaft. Row tabs will be activated based on number of rows selected here.
2	Row Tabs	Selecting an active tab allows the user to define parameters for that row.
3	Bearing	Button opens the bearing selection window for assignment of the bearing in this row position. See section 3.1.4.1 for the bearing selection window.
4	Contact Angle	Contact angle definition for active row. A divergent contact angle extends away from, or diverges, from the spin axis as you traverse in the positive direction along the x-axis.
5	Preload Condition	Radio button to specify the condition at which the row preload (see #6 below) is defined. Un-mounted means the rings are radially free at the specified preload. Mounted means the specified preload includes boundary constraints such as fitup and clamping effects.
6	Row Preload (lbf)	Specifies the preload force applied to the active bearing row. Preload forces are directional and must include the appropriate sign convention. For standard preloading specify a positive preload for divergent contact angles and a negative preload for convergent contact angles.
7	Preload Type	Radio button selection for either rigid or spring type preload. Rigid preloading activates input fields for inner and outer ring clamping forces. Spring preloading activates inputs for the spring rate and snubber gap. <ul style="list-style-type: none"> Clamp forces effect ring diameters per Poisson's theories. Snubber Gap specifies the gap from the preloaded position to a spring hard stop ('snubber'). The gap is always specified as a positive value. <ul style="list-style-type: none"> CAUTION: Applying a zero gap will lock the bearing at the preload condition
8	Add Spacer	Selection of this checkbox places a spacer between the active row and the next row. The drop-down selection will activate to specify the material for the spacer.
9	Row Inputs	The following inputs are entered here: <ul style="list-style-type: none"> Row Location (in): Sets the x-axis position of the active row I.R. Fitup (in): Sets the inner ring fitup (diametral). Fitup is defined as the difference in the shaft O.D. to the free bearing I.D. A positive value indicates interference fits. O.R. Fitup (in): Sets the outer ring fitup (diametral). Fitup is defined as the difference in the free bearing O.D. and the housing I.D. A positive value indicates interference fits. Shaft I.D. (in): Specifies the I.D. of the shaft at the bearing location. For a solid shaft input a zero value. This parameter is used to determine the hoop stiffness of the shaft. Housing O.D. (in): Specifies the O.D. of the housing at the bearing location. This parameter is used to determine the hoop stiffness of the housing. Shaft Temp: Used to specify unique row-wise shaft temperature. Units follow definition from radio button discussed in section 3.1.1. Housing Temp: Used to specify unique row-wise housing temperature. Units follow definition from radio button discussed in section 3.1.1. Coeff. of Ball Friction: Specifies the rolling contact friction coefficient for the active bearing row.

Figure 3.1-6. Bearing Row Inputs

3.1.4.1 Bearing Selection Dialog

The bearing selection dialog is used to assign a specific bearing object from the user database to a row in the system. Refer to the figure below for a full description of the dialog.



#	Title	Description
1	Select Bearing Type	Drop-down menu specifies type of bearing to use. Options are Angular Contact/Radial or 4PT Gothic.
2	Bearing Database List	This list shows all defined bearings in the user's database for the selected bearing type.
3	Parameter Summary	Once a bearing is selected (left mouse click) from the Bearing Database List the parameters of the bearing are shown here.
4	Cancel Button	Pressing 'Cancel' exits the window without selecting anything. The window is closed automatically.
5	Select Button	Pressing the 'Select' button assigns the selected bearing to the row. The window is closed automatically.

Figure 3.1-7. Bearing Selection Dialog

3.1.4.2 Contact Angle Orientation

Contact angle orientation is assigned within the bearing row input region and terminology for convergent and divergent is further illustrated in the figure below.

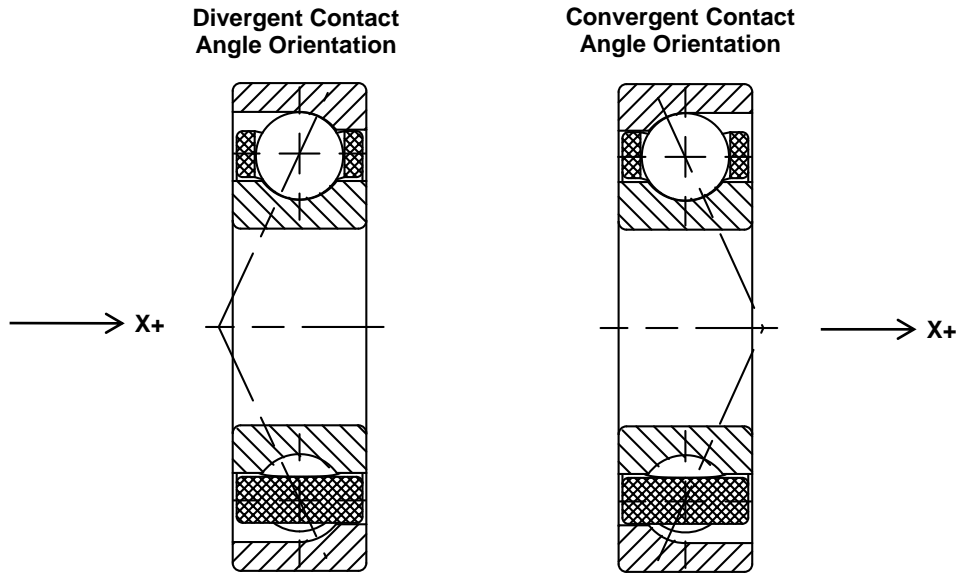


Figure 3.1-8. Convergent and Divergent Contact Angles

3.1.5 Dashboard

The dashboard area (lower portion of the main window) provides helpful user information and is also where completed setups are submitted for analysis. The following figure illustrates the various components of the dashboard area.

#	Title	Description
1	Input Field Description	The input field description bar provides helpful information to the user for each input field. When the user places their cursor in one of the input fields (done by clicking the mouse in a field or pressing the tab button to advance to the next input field) a description about that field appears in the Input Field description bar. Example shown pertains to the external load F_x (lbf) input field.
2	Exit Button	Select this button to safely exit. User will be prompted to save any changes that have occurred to the active setup.
3	Analyze Button	The analyze button is used to submit the user setup for final analysis. Upon completion of an analysis the Results window (reference section 3.3) will automatically display.
4	Display Console Checkbox	Selecting this checkbox will display the console window. See section 3.1.5.1 for further description of the console.

Figure 3.1-9. Dashboard Area

3.1.5.1 Console Window

The console window, which is normally hidden, is used to display solver messages. Information, such as solver progress, is automatically output to this window during each analysis submittal. Selecting the <clear> button at the bottom of the console window will erase all current contents of the console, otherwise text within the console is persistent and will remain available during the entire session.

Common information displayed to the console window includes:

- Convergence criteria (see section 4.2)
- Norm of the residuals for each solver attempt
- Element contact loss at inner race
- Element equilibrium cannot be achieved
- Row expansions due to thermal loads exceed initial preload
- Number of iterations analyzed and if truncation is found when running a Worst-Case Tolerance analysis
- Various step convergence information when running a Flexible Shaft analysis
- Confirmation of convergence

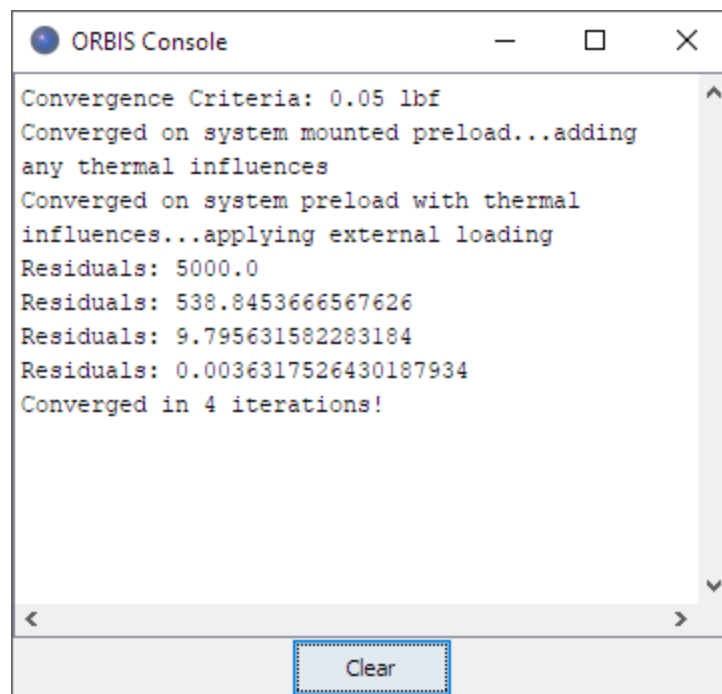


Figure 3.1-10. Console Window

3.1.6 User Menus

User menus are available within the main graphical interface. See the following figure for a detailed description of the available menu options.

#	Title	Description
1	File Menu	<p>The file menu provides the following options:</p> <ul style="list-style-type: none"> • Open: This option generates a file browser for loading an existing ORBIS analysis file (*.jdh). • Save: Used to save all settings in the main window to a file for later use. • Save As: Used to save an existing file to a different name. • Clear Inputs: This option clears all the inputs in the main window and resets any default values. • Exit: Exits the application (user will be prompted to save if there are non-saved changes pending).
2	Tools Menu	<p>This menu provides advanced analysis tools shown below:</p> <ul style="list-style-type: none"> • Batch Process Load Cases: This utility allows the user to load a pre-configured comma delimited file with an unlimited number of load cases to be automatically analyzed. See section 3.1.6.1 for a detailed description of how to use this function. • Shaft Flexibility: Selecting this menu option launches the Shaft Flexibility Analysis dialog. See section 3.1.6.2 for a detailed explanation of this dialog window. • Sensitivity Studies: Selecting this utility launches the Sensitivity Studies window. See section 3.1.6.3 for a detailed explanation of this dialog window. • Tolerance Studies: Selecting this utility launches the Tolerance Analysis window. See section 3.1.6.4 for a detailed explanation of this dialog window. • Dahl Torque Hysteresis: Selecting this menu launches the Dhal Torque Hysteresis window. See section 3.1.6.5 for a detailed explanation of this dialog window. • Enforced Shaft Deflections: Enforced shaft deflections allows the user to specify a known deflected state of the shaft. See section 3.1.6.6 for a full description of this tool.
3	Utilities Menu	<p>This menu provides the following utilities:</p> <ul style="list-style-type: none"> • System Preferences: Selecting this utility launches the System Preferences window. See section 3.1.6.7 for a detailed explanation of this dialog window. • Load License File: Utility to load a new license file. See section 3.1.6.8 for further information. • About: This menu provides an 'About' window to show key parameters regarding the current software version and lease expiration date.

Figure 3.1-11. User Menu's

3.1.6.1 Batch Process Load Cases

This utility allows the user to load a comma delimited text file (*.csv) for batch processing. The text file will contain external load cases to be analyzed against the current system setup. When this utility is run, each load case defined in the external file will be copied into Load #1 within ORBIS and analyzed. The routine runs in silent mode and automatically saves a result file for each load case to the directory where the *.csv file is located. Each result file is named bases on the load case number. The figure below describes how to perform a batch process.

STEP 1:
Create external file with load cases to be analyzed. File must be formatted as follows (see table and corresponding example with Microsoft Excel®):

- Each row contains a unique load case
- Columns contain load components and load location as shown in table below

	Fx	Fy	Fz	Fyy	Fzz	Location
Case 1 →	100	500	0	0	1000	1.5
Case 2 →	200	350	0	0	980	1.5
...	300	275	0	0	880	1.5
Case n →	125	600	0	0	1200	1.5

STEP 2:
Save file as a .csv format.

STEP 3:
Within ORBIS, launch the Batch Process utility from the tools menu. The dialog below will be shown. Select 'OK' button and a file browser will allow you to select the defined *.csv file from STEP 2.

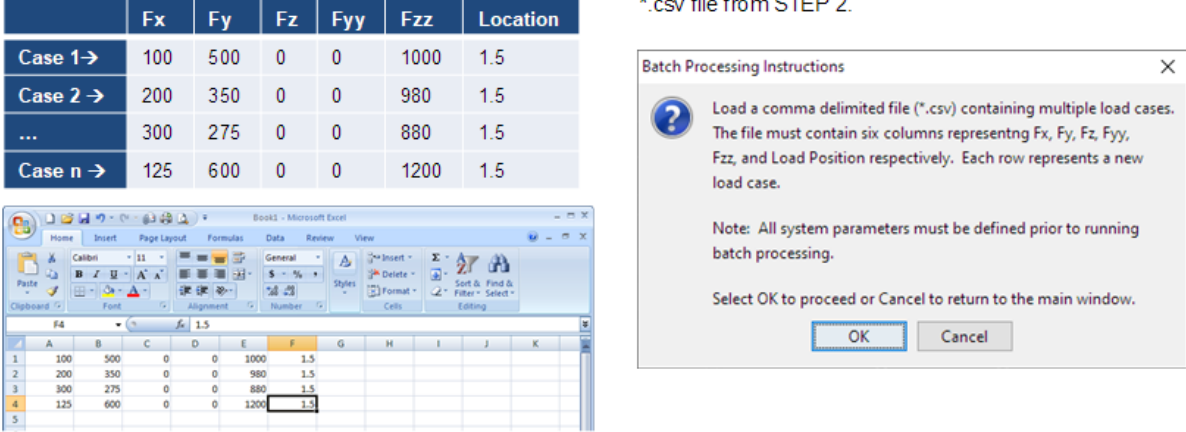


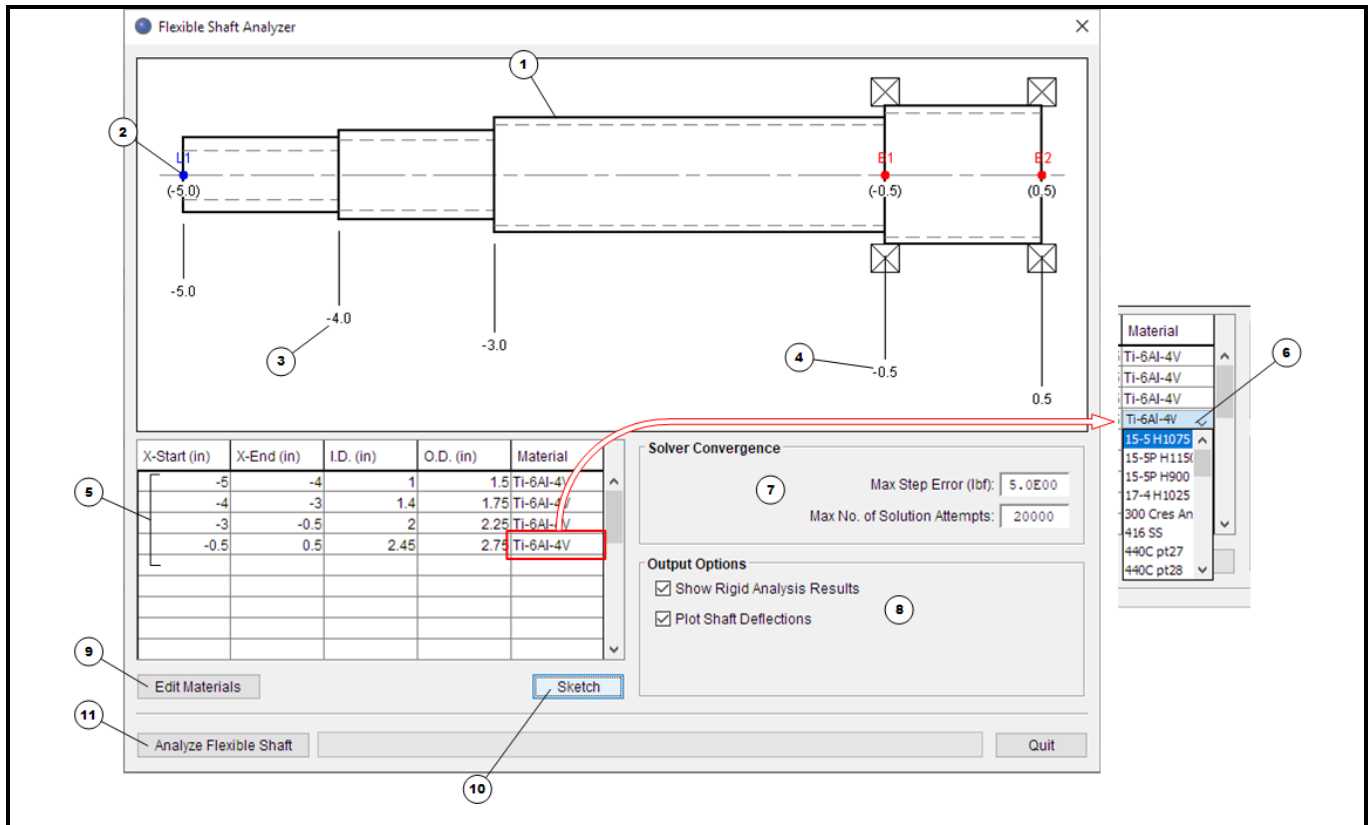
Figure 3.1-12. Batch Processing Load Cases

3.1.6.2 Shaft Flexibility

The Flexible Shaft Analyzer is used to account for elastic compliance of the bearing shaft. ORBIS models the shaft with classical Timoshenko beam element formulations that account for both bending and shear deflections in the shaft. The interface allows shaft definition of up to 25 unique circular beam elements; each of which may be defined with unique section dimensions and/or materials. See figure below for descriptions of the flexible shaft window.

To perform an analysis that considers shaft flexibility the user completes their system setup within the main window as if performing a standard rigid analysis. Once the system is setup the user selects the 'Shaft Flexibility' option from the Tools menu to open the Flexible Shaft Analyzer window (as shown in the figure below). Here the user defines their shaft elements, reviews their final setup and submits the final analysis.

If the initially defined rigid system has constant section dimensions, determined by validating all defined bearing I.D.'s and shaft I.D.'s are constant, ORBIS will prepopulate one shaft element within the Flexible Shaft Analyzer window that extends through the complete system. The user may override this assumption by editing the table of shaft elements. For cases where the bearing I.D.'s or shaft I.D.'s are not constant throughout the system, no prepopulated elements are provided and the user will need to define shaft elements that extend through all bearing and load locations.



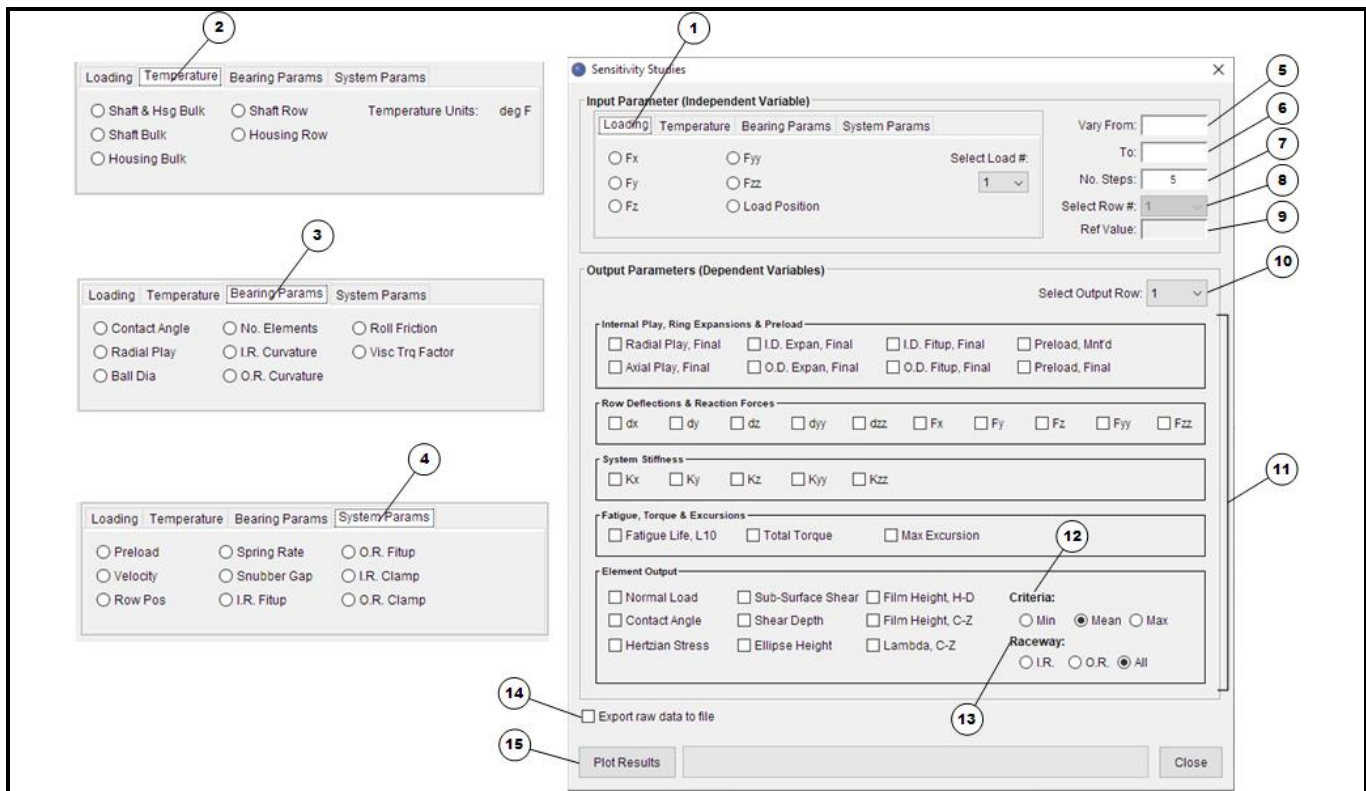
#	Title	Description
1	Shaft Sketch	An engineering sketch is provided for the user defined setup.
2	Load Points	All user defined load points are labeled (L1, L2, etc) with their x-axis coordinate.
3	Element Dimensions	Each user defined shaft element is sketched and dimensioned along the length (x-axis).
4	Bearings	All user defined bearings are sketched and x-axis coordinates are shown.
5	Shaft Element Definition Table	All shaft elements are defined here. All shaft elements are cylindrical and each successive element must start where the previous element ended. Additionally, shaft elements must exist for the entire span of the defined system, consisting of the left-most bearing row or load location to the right-most bearing or load location.
6	Element Material Properties	Unique element properties are assigned in the material column. This column is directly linked to the Material database and materials are easily assigned by selecting from a drop-down menu (as shown on right).
7	Solver Convergence	You can modify the maximum allowable step error and the maximum number of solution attempts. The solver has an adaptive method that continuously reduces the load increment (step size) until either the step error is satisfied or the maximum number of solution attempts has been exceeded.
8	Output Options	Output options exist for showing the rigid shaft analysis results (same results one would get with a normal ORBIS analysis from the main window) and whether shaft deflections should be plotted.
9	Edit Materials Button	Launches the Material Database editor. When done editing materials the user will be returned to the Shaft Flexibility window.
10	Sketch Button	Once the user has input the shaft element definitions selecting the 'Sketch' button updates the sketch window.
11	Analyze Flexible Shaft Button	Runs the flexible shaft analysis and produces a results window.

Figure 3.1-13. Flexible Shaft Window

3.1.6.3 Sensitivity Studies

The Sensitivity Studies utility enables rapid bearing design and quick solutions to common ‘what if’ scenarios. This utility allows the user to vary almost any input parameter (independent parameter) and plot them against any output parameter (dependent parameter). The Sensitivity Study dialog is accessed from the ‘Tools’ menu (see section 3.1.6) on the main window. This utility requires a complete analysis definition within the main window and also requires the dynamic analysis mode be selected.

The following subsection discusses interaction options for the plot windows generated from all sensitivity study runs. In addition to the generated plots, the user can export the raw data used to generate the plots for post processing. The data is saved in a delimited text file that can be easily imported into programs such as Microsoft Excel®.



#	Title	Description
1	Loading Input Variables	Loading input variables include each load component and the load position. Furthermore, there is an option to select which load number is used.
2	Temperature Input Variable	Temperature input variables include options for bulk (shaft and housing), bulk shaft, bulk housing, or the row specific component. For row specific components see item 8. Temperature units are also shown for reference.
3	Bearing Parameter Input Variables	Bearing input variables include contact angles, radial play, ball diameter, number of elements, curvatures, rolling friction and viscous torque factor. See item 8 for row specification options.
4	System Parameter Input Variables	System input parameters include preload, velocity, row location, spring rate, snubber gap, IR/OR fitups, and IR/OR clamp loads. See item 8 for row specification options
5/6	Vary From/To	Specifies the start and end value of the perturbation for the selected input variable
7	No. Steps	ORBIS will use the specified variable limits (Vary From/To) and break it into equal steps based on this input value. Each step is a solution point in the generated plots.
8	Selected Row #	This drop-down is active depending on input variable. If input variable can be row specific the drop-down is active. Available options include each of the defined bearing rows in the system or the option to apply to all rows simultaneously.
9	Ref Value	This is a non-editable field that provides the nominal value of the selected independent parameter. This is provided for user reference.
10	Select Output Row	Specifies which output row to be used to plot output (dependent variable).
11	Output Variable Selections	Checkbox selection of all possible output variables. User may select any number of output variables to analyze. Orbis will provide a separate plot window for each output variable.
12	Criteria	Element output allows criteria options: minimum, mean or maximum to be specified
13	Raceway	Element output allows raceway options: I.R., O.R. or both
14	Export raw data to file	Selecting this checkbox will allow the user to save the raw plot data to a delimited text file.
15	Plot Results Button	Button to perform analysis and plot final results. If user selected 'Export raw data to file' checkbox the user will be presented with 'File Save' dialog to specify a file name and directory for the data file prior to plotting the data.

Figure 3.1-14. Sensitivity Studies Dialog

3.1.6.3.1 Sensitivity Plot Windows

The plot windows generated from a sensitivity study are interactive. Separate plot windows are generated for each dependent variable selected in the sensitivity dialog. See the following figure for a description of user options within the plot windows.

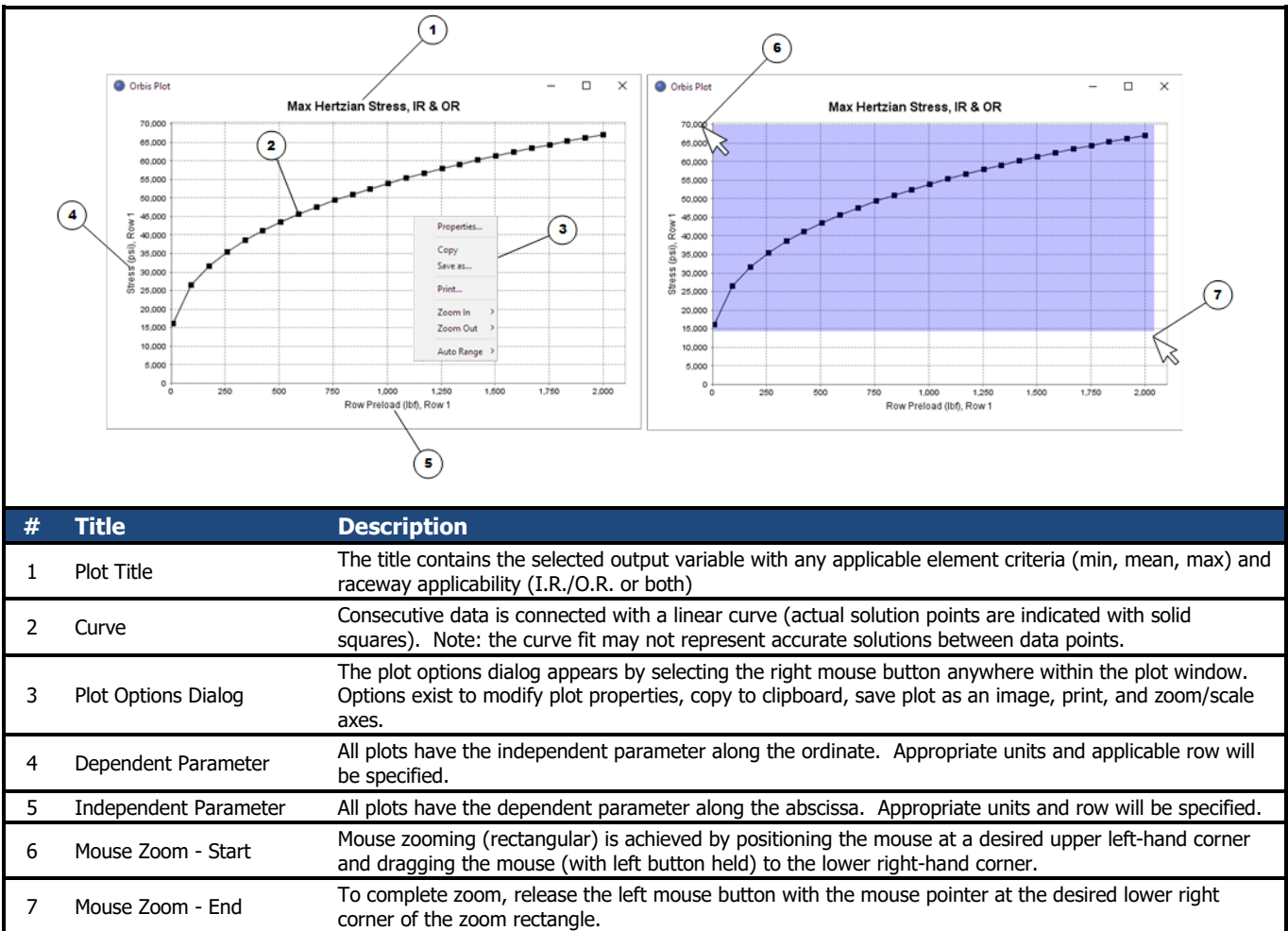
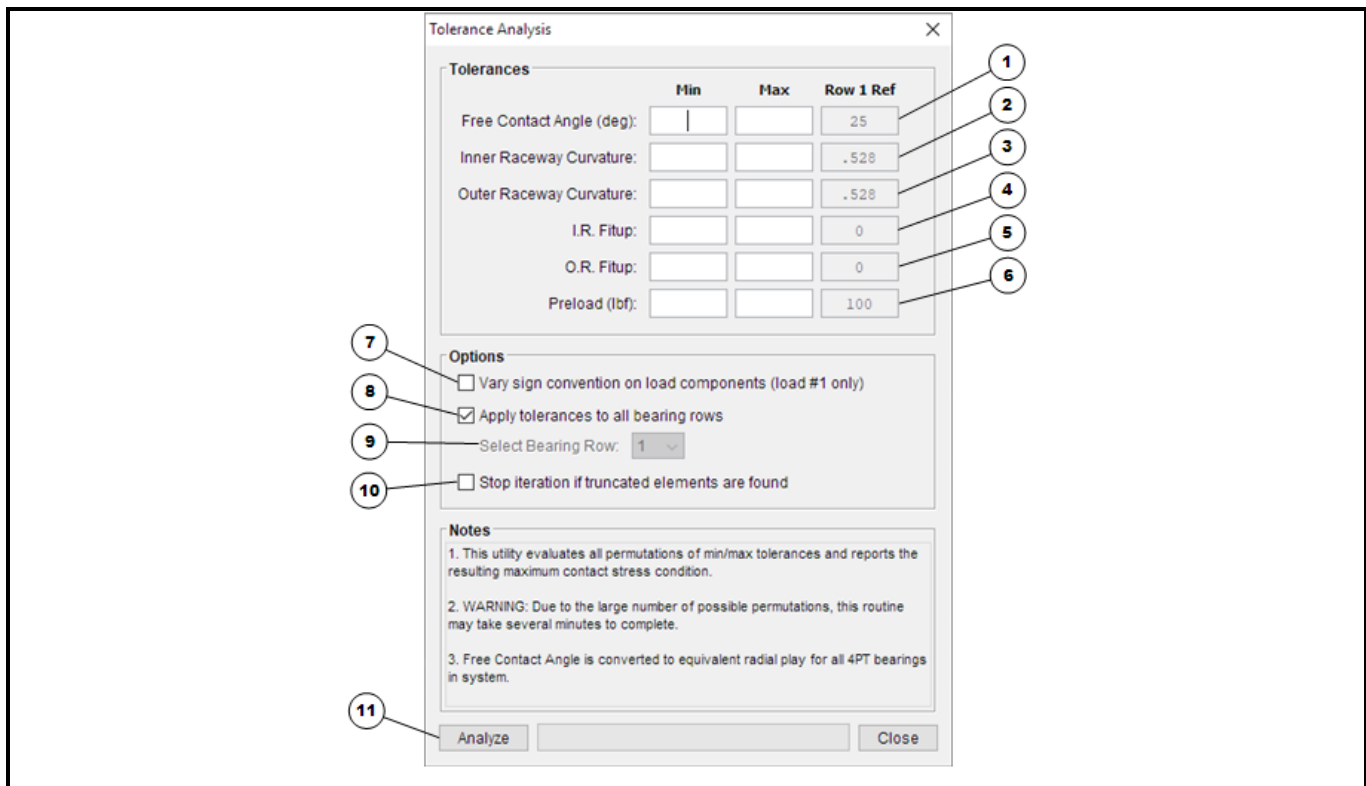


Figure 3.1-15. Plot Windows

3.1.6.4 Tolerance Studies

ORBIS can perform tolerance studies of key bearing parameters with the Tolerance Analysis utility. This utility iteratively solves all permutations of user specified tolerances, min and max conditions, and provides the combination causing worst case Hertzian contact stress in a results window. Additionally, the utility can check for truncation and will display a result window with the truncated conditions. Reference the following figure for a description of the Tolerance Studies utility.



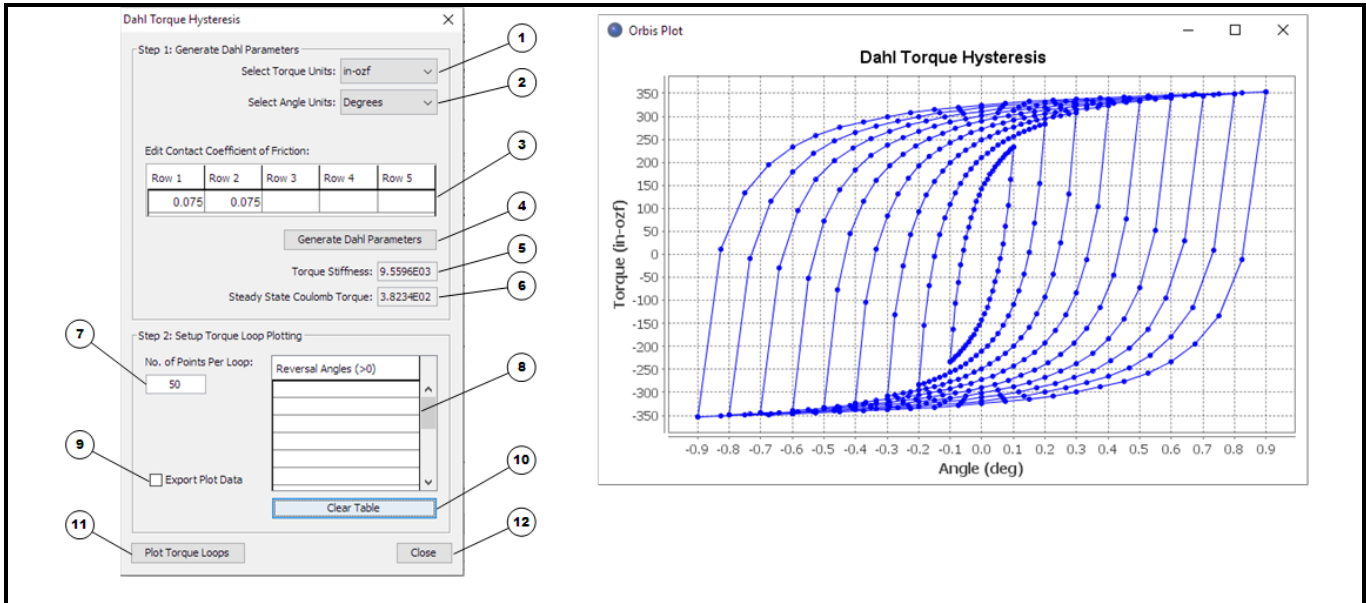
#	Title	Description
1	Free Contact Angle (deg)	Specify min and max free contact angles in degrees.
2	Inner Raceway Curvature	Specify min and max inner raceway curvatures (ratio of raceway radius to ball diameter).
3	Outer Raceway Curvature	Specify min and max outer raceway curvatures (ratio of raceway radius to ball diameter).
4	I.R. Fitup	Specify min and max inner ring fitup (positive value is interference)
5	O.R. Fitup	Specify min and max outer ring fitup (positive value is interference)
6	Preload	Specify min and max preload (always as a positive value). Orbis will correct sign based on contact angle orientation and the assumption that the preload should load through the contact angle.
7	Vary Sign Convention	Checkbox to specify if sign convention should be varied on all load components. Selecting this checkbox will run all permutations of positive and negative load components.
8	Apply Tolerances to all rows	Checkbox to specify how tolerances are applied to the system. Selecting this checkbox causes all bearing rows to have specified tolerances analyzed. Un-selecting this checkbox activates item 7 below and tolerances are only applied to selected bearing row.
9	Row Selection Drop-down	When active the drop-down menu is used to specify which bearing row to apply the specified tolerances.
10	Truncation Check	Checkbox to specify if the search algorithm should stop if any element is found to contain truncation. If selected and a set of tolerance conditions results in truncation then the first case found will be shown in a results window.
11	Analyze Button	Button to begin analysis of tolerances.

Figure 3.1-16. Tolerance Studies Dialog

3.1.6.5 Dahl Torque Hysteresis

The Dahl Torque utility is used to analyze the torsional stiffness (torque versus angle) of the bearing system during startup or direction reversal. This phenomenon occurs through small finite angles of rotation, often most apparent when direction of rotation is reversed, at speeds sufficiently slow such that viscous drag is negligible. The utility provides quick inspection of the reversing torque slope and steady state torque. Additionally, the utility can quickly generate small angle hysteresis loops for both graphical plot inspection and data export. The analysis procedures of this utility follow those set forth by Todd and Johnson (1986).

To perform a Dahl torque analysis the user configures their system within the main window as if performing a standard analysis run. Once the system is completely defined the user selects the ‘Dahl Torque Hysteresis’ option from the Tools menu. A new dialog will appear, as shown in the figure below, with various configuration options.



#	Title	Description
1	Torque Units	Drop-down menu allows selection from units of in-ozf, in-lbf, ft-lbf, gm-cm, N-cm or N-m. All subsequent results will be in selected units.
2	Angle Units	Drop-down menu allows selection from units of degrees or radians. All subsequent results will be in selected units.
3	Coefficient of Friction	Tabular input of contact coefficient of friction for each bearing row. Modifications of this parameter will proportionally alter the resulting steady state hysteresis torque.
4	Generate Dahl Parameters Button	Selecting button generates torque stiffness and steady state torque results. See #'s 5 & 6 below.
5	Torque Stiffness	Result has units of torque divided by angle where units are as specified above.
6	Steady State Coulomb Torque	Steady State friction torque value with units as specified above. This can be adjusted by altering the contact coefficients of friction.
7	No. of Points Per Loop	Specifies the number of solution points per hysteresis loop. Points are always equally spaced from +/- reversal angles specified.
8	Reversal Angles	Table allows multiple reversal angles to be generated in the plot output. Units are as specified above. Angles must be entered as positive values greater than zero. A separate loop is generated for each defined table entry.
9	Export Plot Data Checkbox	Selecting this checkbox creates a save dialog once the Plot Torque Loops button is pressed. The user will then be able to save the data from the plot to a delimited text file for post processing.
10	Clear Table Button	Button simply clears all reversal angle data within the table.
11	Plot Torque Loops Button	Generate torque loop plot based on all above user settings. Generated plot window has all standard zoom and right-click context menu options as discussed in Figure 3.1-15.
12	Cancel Button	Button closes the dialog and returns to the main window

Figure 3.1-17. Dahl Torque Hysteresis Utility

3.1.6.6 Enforced Shaft Deflections

This utility enforces a prescribed deflection on the shaft and generates a full set of results. Specified deflections are relative to the mounted and preloaded state. Any defined external loading will be ignored. The figure below shows the dialog for defining enforced shaft deflections.

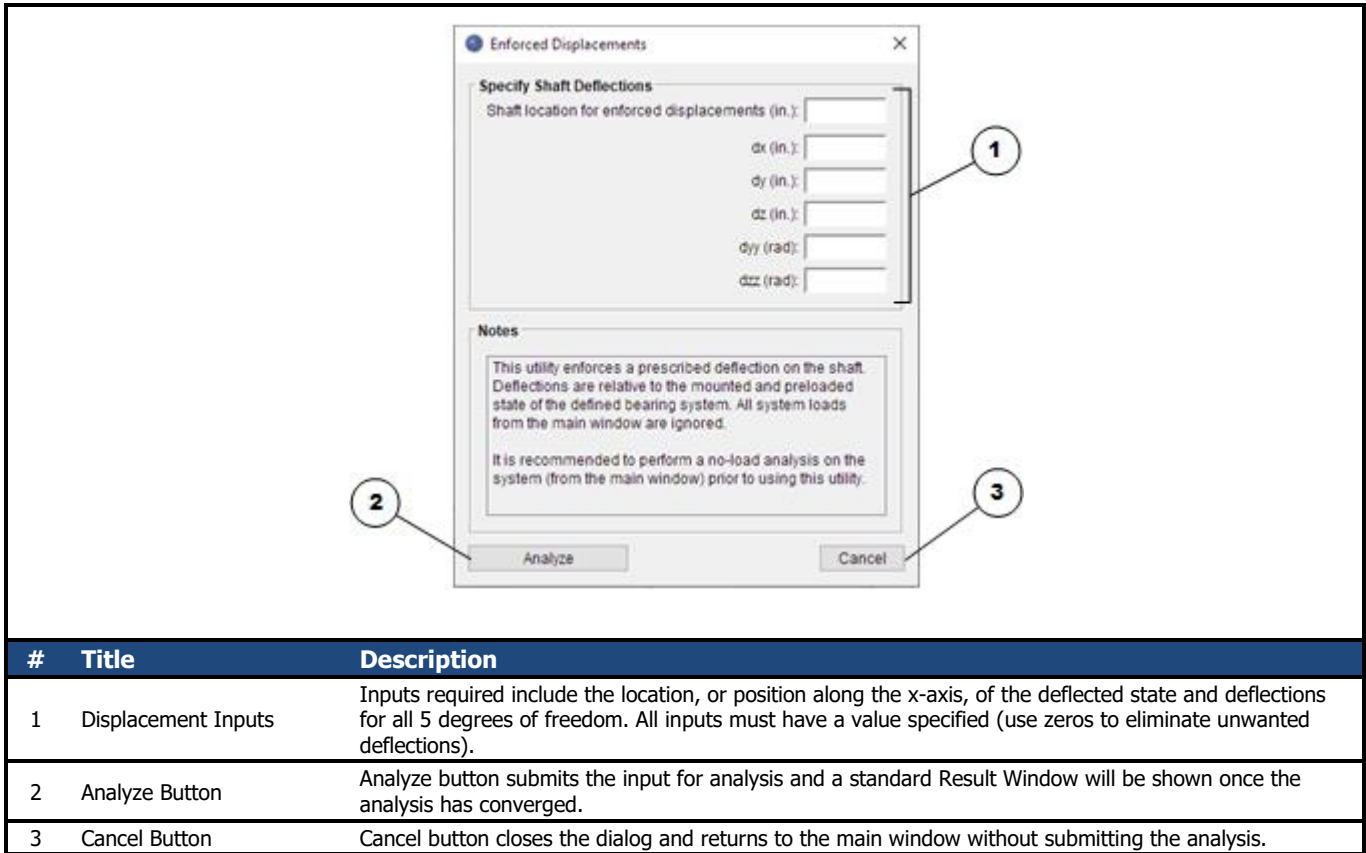
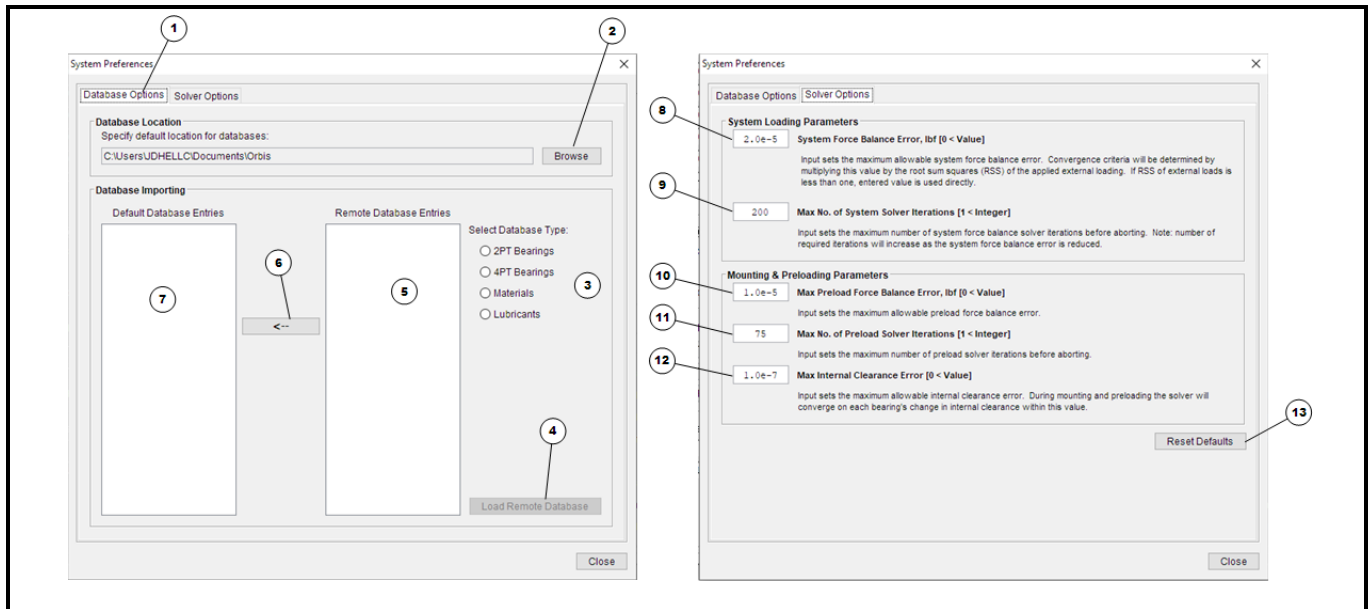


Figure 3.1-18. Enforced Displacements

3.1.6.7 System Preferences

The system preferences dialog is available from the utilities menu (reference 3.1.6). These preferences are persistent; meaning they remain in effect each time the user launches and runs ORBIS, until changed from this dialog.

System preferences settings allow control of the database location, importing external database entries, and setting solver convergence criteria. Options available are shown in the figure below.



#	Title	Description
1	Tabbed Pane Selection	Each system preference category is accessed by selecting the appropriate tab.
2	Database Location	This area contains the folder location to the default database files. The browse button brings up a folder selection dialog that allows the user to this location.
3	Select Database Type	To perform a database import, or simply view contents of a database, the user must select which type of database to be displayed. Once the type is specified the two scroll panes (see #6 & #7) populate with the appropriate entry names.
4	Load Remote Database	Select this button to bring up a folder selection dialog. Within the folder selection dialog, the user will navigate to a desired database. Note: this button is only active once a database type is specified.
5	Remote Database Entries	Displays entries from remote database. Single entry selection is achieved by mouse selection (left click) on desired entry. Multiple entry selection is achieved by two methods: for consecutive entry selection select the first desired entry and shift+click the last consecutive entry, for non-consecutive entry selection select each entry while holding the control key.
6	Import button	Selecting this button imports the selected remote database entries into the default database. The database does not allow duplicate entries and will check each import entry for existing names within the default database. If duplicates are found the user will be asked to confirm overwriting or skip the duplicate import.
7	Default Database Entries	Displays the entries from the user's default database.
8	System Force Balance Error	This entry specifies the allowable system force balance error for solver convergence (Norm of the residuals). Acceptable values are between zero and one exclusive [0<value<1]. The system force balance convergence criteria will be determined by multiplying this value by the RSS of the applied external loading. In the case where the RSS of the external loading is less than one this value will be used directly. For example, if the RSS of the external loading is 1,000 lbf and the system force balance error is 1.0E-5 (default value), then the system will converge to equilibrium within 0.01 lbf.
9	Max Solver Iterations	Specifies the maximum number of numerical solver iterations before aborting.
10	Max Preload Error	This entry specifies the maximum allowable force error for determining the preloaded state. Acceptable values are greater than zero [0<value].
11	Max Preload Iterations	Specifies the maximum number of numerical solver iterations on preload convergence before aborting.
12	Max Internal Clearance Error	Specifies the maximum allowable internal clearance error for convergence, in inches. Acceptable values are greater than zero [0<value]. During mounting and preloading routines the solver will converge on each bearing row's change in internal clearance within this value.
13	Reset Defaults	Select this button to restore all solver options to their default values.

Figure 3.1-19. System Preferences Dialog

3.1.6.8 Load License File

The Load License File option is available from the utilities menu (reference 3.1.6). This function is used to update the user license via a unique file supplied by the developer. Launching this utility will show a dialog per the following figure. To properly update the user license, the license file must be placed in the ORBIS installation directory and the USB dongle must be inserted into the computer before pressing the ‘Proceed’ button on the dialog. Note: the dialog shows the path where the license file should be located.

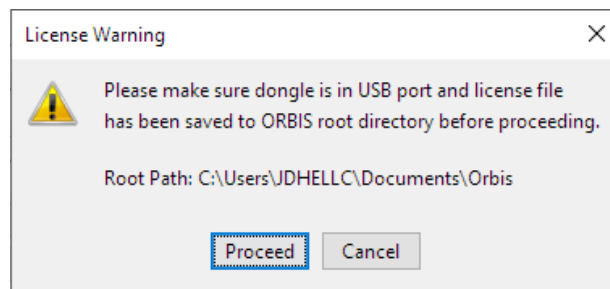


Figure 3.1-20. Load License File Dialog

3.1.6.9 About

The About dialog is available from the utilities menu (reference 3.1.6). This dialog is purely informational and shows the user’s license expiration date, the product version and the dongle serial number.

3.2 Database Editors

ORBIS uses bearing, material and lubricant databases to define the majority of input parameters required in a bearing analysis. Once the user has defined their database entries they simply assign them to their analysis setup via drop-down menus within the main window.

Database editors allow the user to view, modify, add and delete bearing, material, and lubricant database definitions. The database editors are accessed from the buttons within the System Display area (see section 3.1.3).

Database definitions are stored in four specific files: Bearings.dat, Bearings4PT.dat, Materials.dat and Lubricants_V2.dat. These files are typically located within the ORBIS installation directory. However, database files can be placed anywhere, such as a shared network drive. In the case where multiple users have access to a common network drive it is recommended that a common set of database files be used for all users. See the System Preferences dialog (section 3.1.6.7) for details on how to import entries from external databases or change the default location of the database files.

It is important to maintain databases within ORBIS. As users begin to create and save various analyses the saved files depend upon database entries being available upon next use. For example, suppose a user creates a material definition titled “440C” and then defines a bearing named “XYZ Bearing” that uses the 440C material for the inner/outer rings and balls. Every time the user wants to run an analysis with XYZ Bearings ORBIS will search the material database for a valid definition named “440C.” If this name is not found ORBIS will generate an error message telling the user the material could not be found. This same situation occurs for all database entries and any user saved analysis setup files.

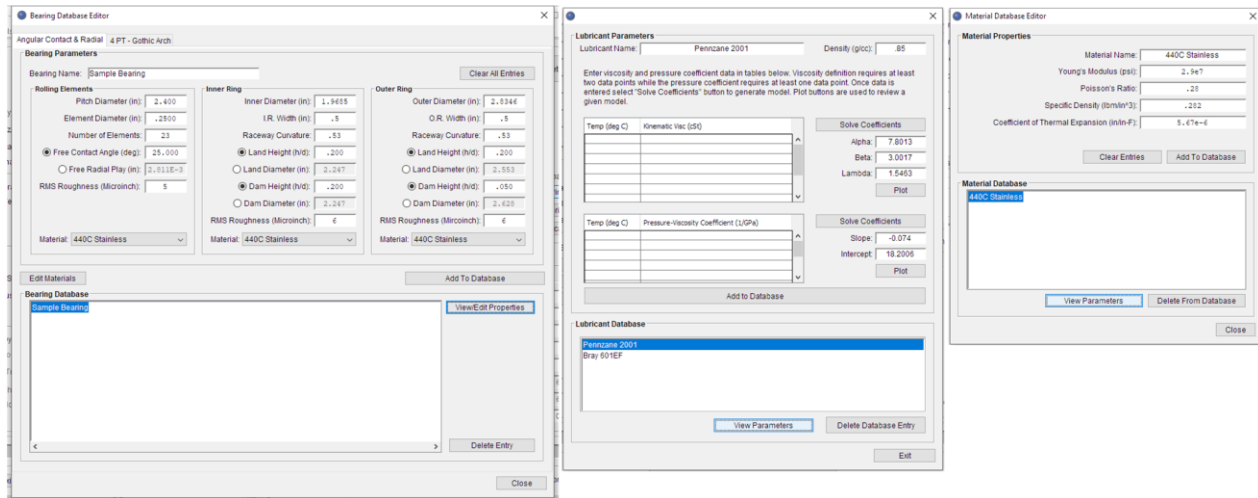
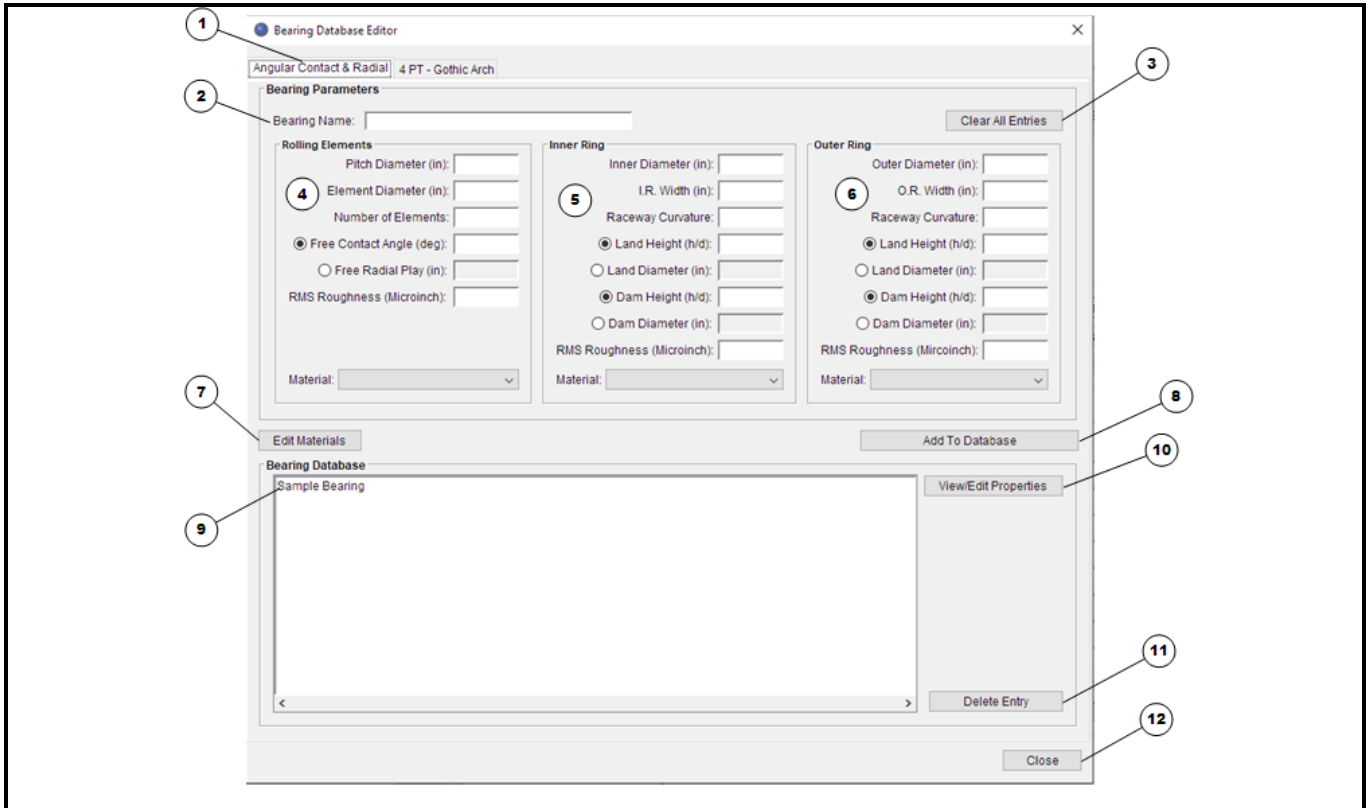


Figure 3.2-1. Database Editors

3.2.1 Bearing Database Editor

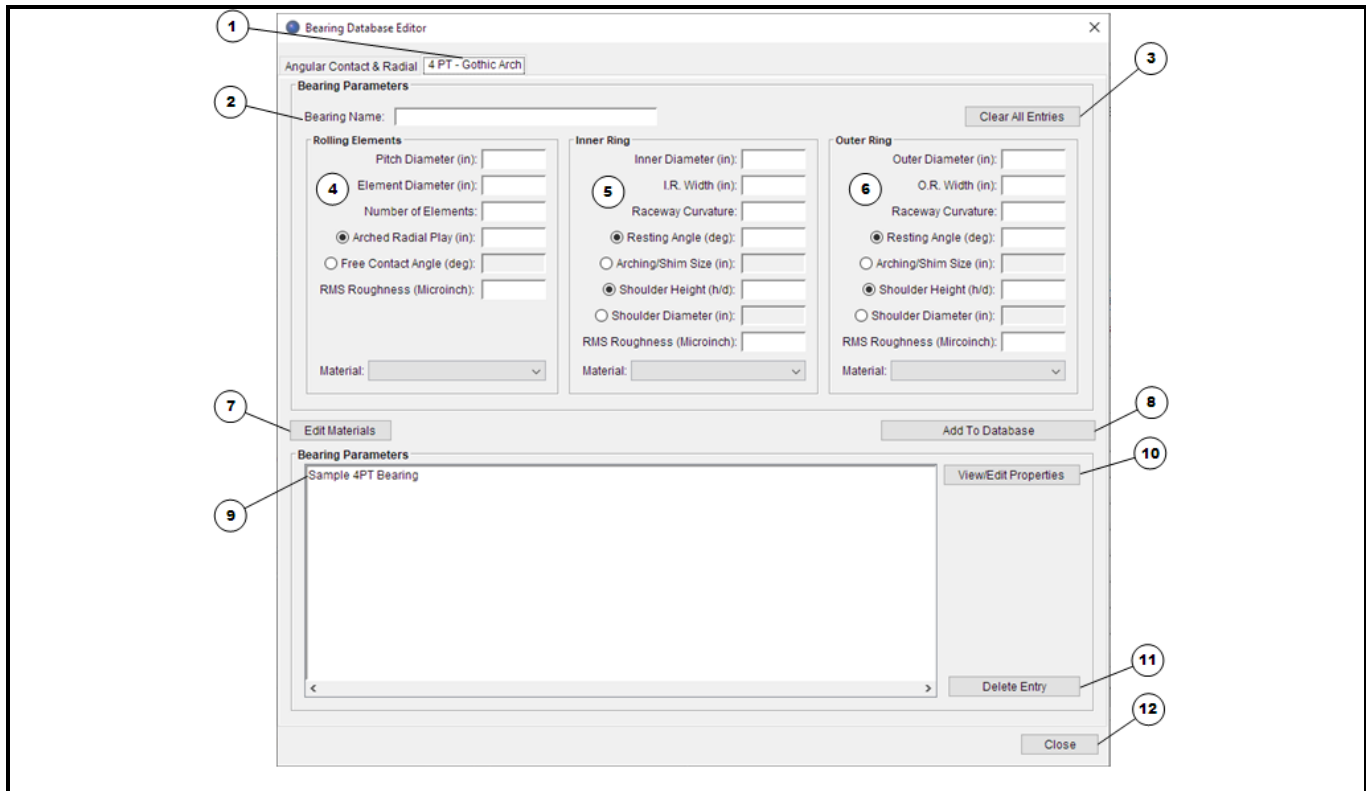
The Bearing Database Editor allows the user to add, edit, view or delete bearing objects. See the following figures for a detailed description of the editor. Note that certain parameters may be defined in more than one way. For example, the free contact angle can be derived from the radial play, ball diameter and curvature ratios or input directly.

To edit existing database entries simply select the existing database entry and click the 'View Parameters' button. All entry fields are populated and the user can then make their changes. To save the changes use the 'Add To Database' button. ORBIS will then ask to verify you want to overwrite the old entry with the new one. If you want to keep the old entries you must rename the new entry with a unique name.



#	Title	Description
1	Bearing Type Tabs	Bearing type options include Angular Contact/Radial or 4PT Gothic Arch. See section 3.2.1.1 for a description of the Angular Contact/Radial type bearing.
2	Bearing Name	Text field for specifying the name of the bearing. The name must be unique (ORBIS will not allow duplicates and will warn the user if a new bearing has the same name as an existing database entry).
3	Clear All Entries Button	Clears all current input field entries. This does NOT clear the database entries.
4	Rolling Element Inputs	Rolling element inputs consist of the following required parameters. Reference Figure 3.2-4 for terminology. <ul style="list-style-type: none"> • Pitch Diameter (in) • Element Diameter (in) • Number of Elements • Free Contact Angle (deg) or Free Radial Play (in) • RMS Roughness (micro-inch) • Material
5, 6	Inner Ring & Outer Ring	Inner and outer rings consist of the following required parameters. Reference Figure 3.2-4 for terminology. <ul style="list-style-type: none"> • Inner/Outer Diameter (in) • I.R./O.R. Width (in) • Raceway Curvature • Land Height (h/d) or Land Diameter (in). See Figure 3.2-5 for terminology. • Dam Height (h/d) or Dam Diameter (in). See Figure 3.2-5 for terminology. • RMS Roughness (Microinch) • Material
7	Edit Materials Button	Opens the Material Database Editor. Once the material database editor is closed the user will be brought back to the Bearing Database Editor.
8	Add To Database Button	Commits the specified input entries into the database under the bearing name specified. A warning will occur if the specified bearing name already exists within the database.
9	Database Entry	This area shows listing of all defined database entries.
10	View/Edit Properties Button	Once a database entry is selected within the database window, selecting this button will populate the bearing parameters into the input fields.
11	Delete Entry Button	With a database entry selected, this button will permanently delete the entry from the database.
12	Close Button	Closes the Bearing Database Editor.

Figure 3.2-2. Bearing Database Inputs, Angular Contact & Radial Bearings



#	Title	Description
1	Bearing Type Tabs	Bearing type options include Angular Contact/Radial or 4PT Gothic Arch. See section 3.2.1.3 for a description of the 4PT Gothic Arch type bearing.
2	Bearing Name	Text field for specifying the name of the bearing. The name must be unique.
3	Clear All Entries Button	Clears all current input field entries. This does NOT clear the database entries.
4	Rolling Element Inputs	Rolling element inputs consist of the following required parameters. Reference Figure 3.2-4 and Figure 3.2-6 for terminology. <ul style="list-style-type: none"> • Pitch Diameter (in) • Element Diameter (in) • Number of Elements • Free Contact Angle (deg) or Arched Radial Play (in) • RMS Roughness (micro-inch) • Material
5, 6	Inner Ring & Outer Ring	Inner and outer rings consist of the following required parameters. Reference Figure 3.2-4, Figure 3.2-5, Figure 3.2-6, and Figure 3.2-7 for terminology. <ul style="list-style-type: none"> • Inner/Outer Diameter (in) • I.R./O.R. Width (in) • Raceway Curvature • Resting Angle (deg) or Arching/Shim Size (in). • Dam Height (h/d) or Dam Diameter (in). See Figure 3.2-5 for terminology. • RMS Roughness (Microinch) • Material
7	Edit Materials Button	Opens the Material Database Editor. Once the material database editor is closed the user will be brought back to the Bearing Database Editor.
8	Add To Database Button	Commits the specified input entries into the database under the bearing name specified. A warning will occur if the specified bearing name already exists within the database.
9	Database Entry	This area shows listing of all defined database entries.
10	View/Edit Properties Button	Once a database entry is selected within the database window, selecting this button will populate the bearing parameters into the input fields.
11	Delete Entry Button	With a database entry selected, this button will permanently delete the entry from the database.
12	Close Button	Closes the Bearing Database Editor.

Figure 3.2-3. Bearing Database, 4PT Gothic Arch Bearings

3.2.1.1 Angular Contact and Radial Bearing Terminology

Figure 3.2-4 illustrates the terminology for an angular contact bearing. The radial/deep groove/Conrad bearing shares all of these parameters but typically the free contact angle is replaced by the parameter for radial play. ORBIS allows either value as an input.

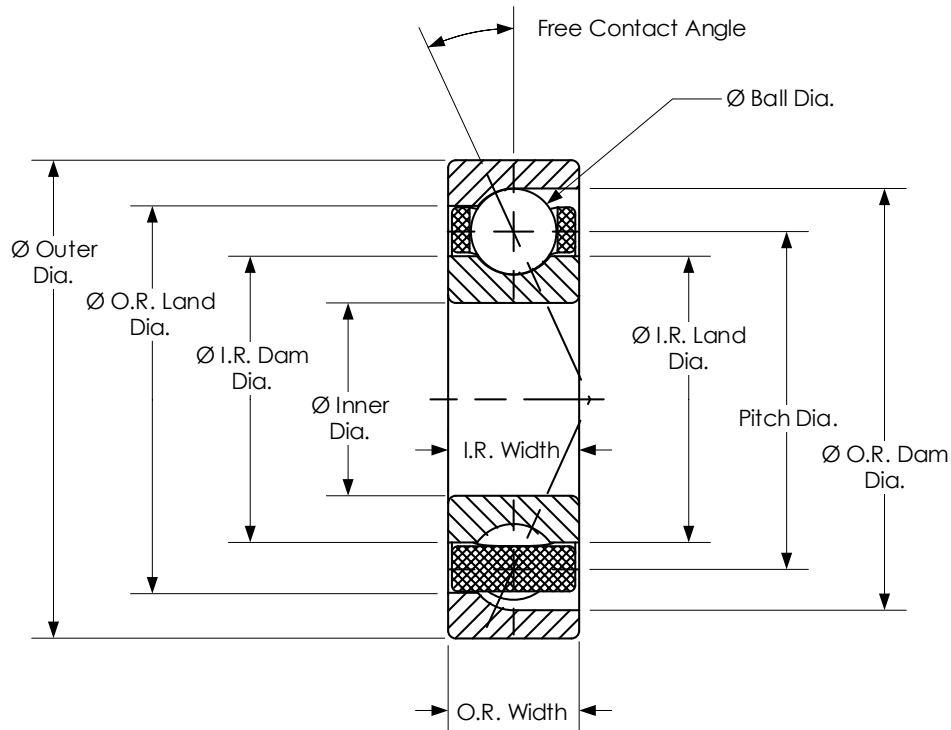


Figure 3.2-4. Angular Contact (2PT) Terminology

3.2.1.2 Shoulder Height Definitions

Land and dam heights can be defined as h/d ratios or diameters. The figure below illustrates the land and dam heights (h). As shown, the term ‘dam’ refers to the non-contacting shoulder and the term ‘land’ refers to the contacting side of the raceway.

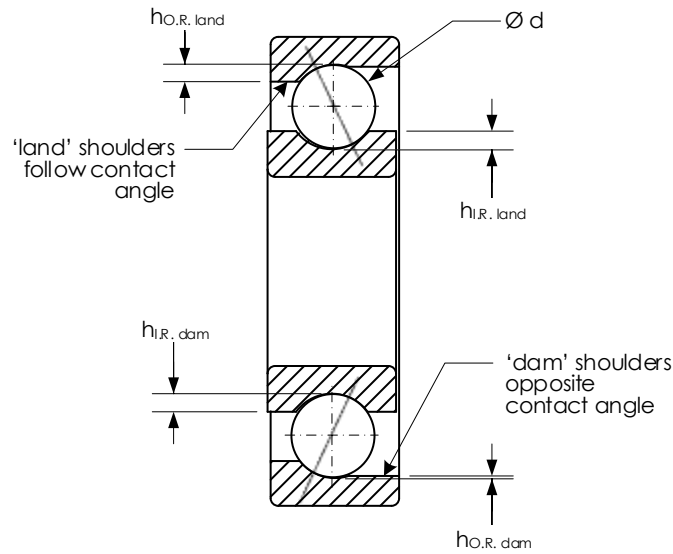


Figure 3.2-5. Shoulder Height Definitions for h/d Values

3.2.1.3 4PT Gothic Arch Bearings

The easiest way to describe a gothic formed raceway is by splitting a deep groove bearing down the middle of the raceway, removing equal amounts of material from the two halves of the splitting faces, and abutting the raceways back together (reference Figure 3.2-6). The removed material is called the arching/shim size.

To define a 4PT Gothic Arch bearing in ORBIS requires a few additional parameters compared to a standard deep groove bearing. In particular, the arched radial play or free contact angle must be defined along with the shim size or resting angle for both the inner and outer races. Figure 3.2-7 shows the definition of the arched radial play, resting angle and free contact angle.

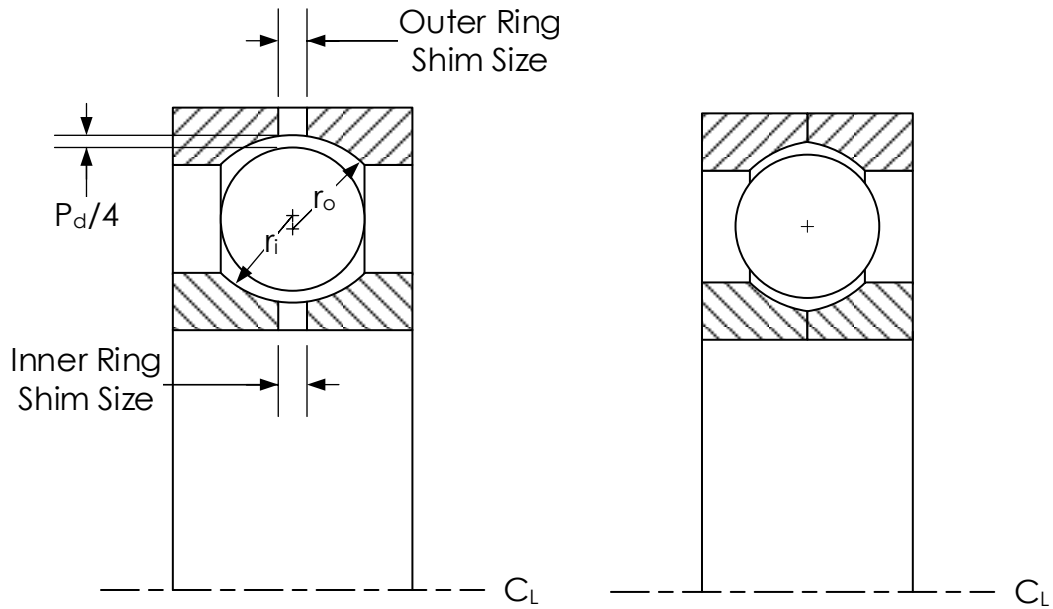


Figure 3.2-6. Gothic Arch (4PT) Arching/Shim Terminology

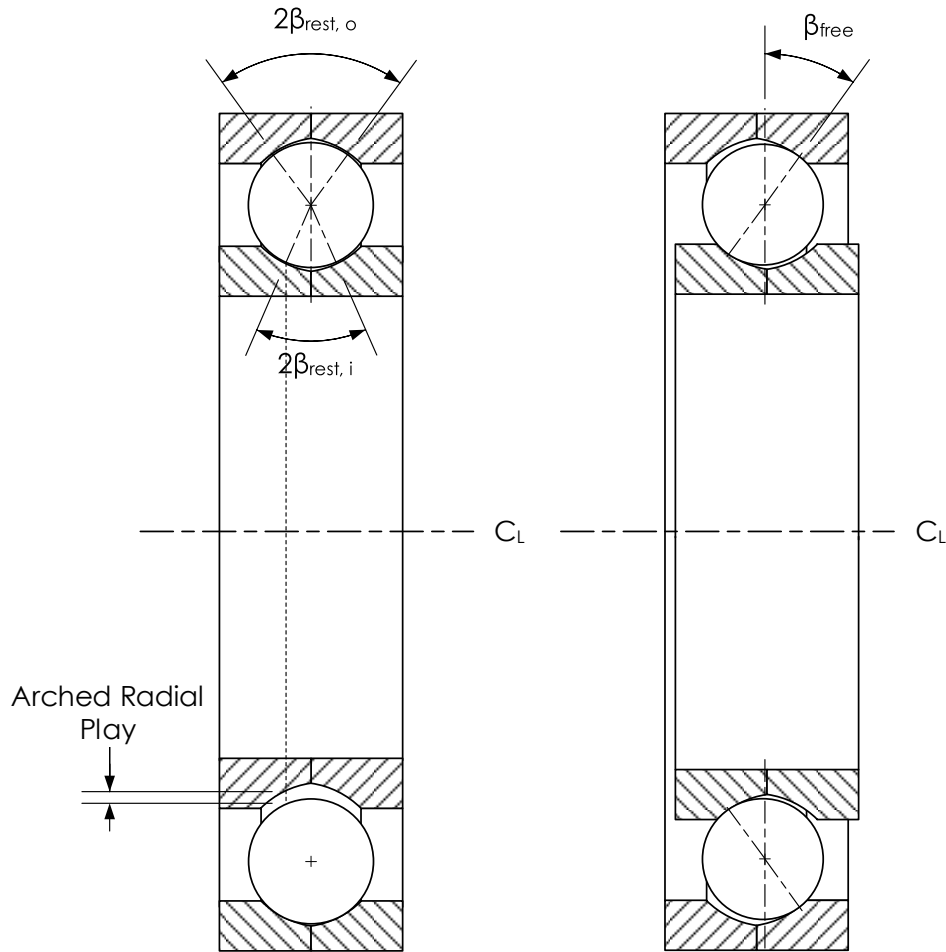
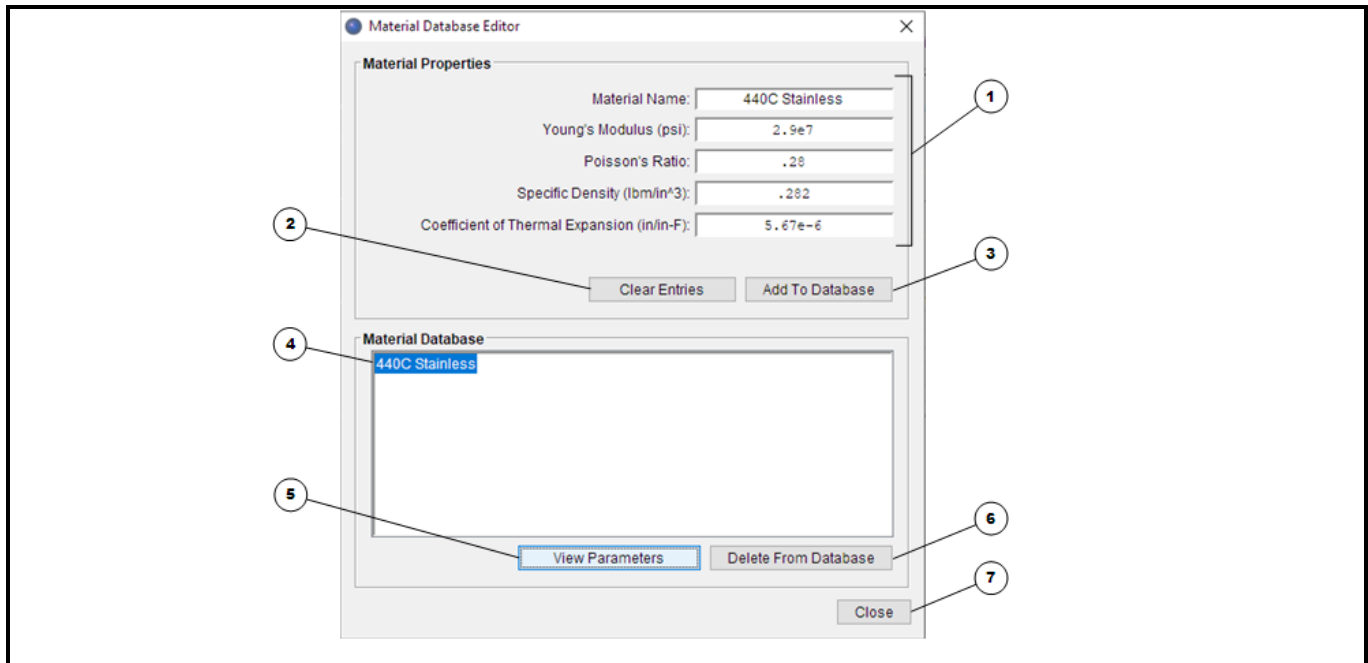


Figure 3.2-7. 4PT Radial Play, Resting Angle and Free Contact Angle Terminology

3.2.2 Material Database Editor

The material database editor allows the user to define their materials. Since material definitions are needed for both bearing definitions and shaft/housing definitions it is recommended that the user initially take the time to define all of their most widely used materials before proceeding with analysis setups. See the following figure for a detailed description of the material editor.

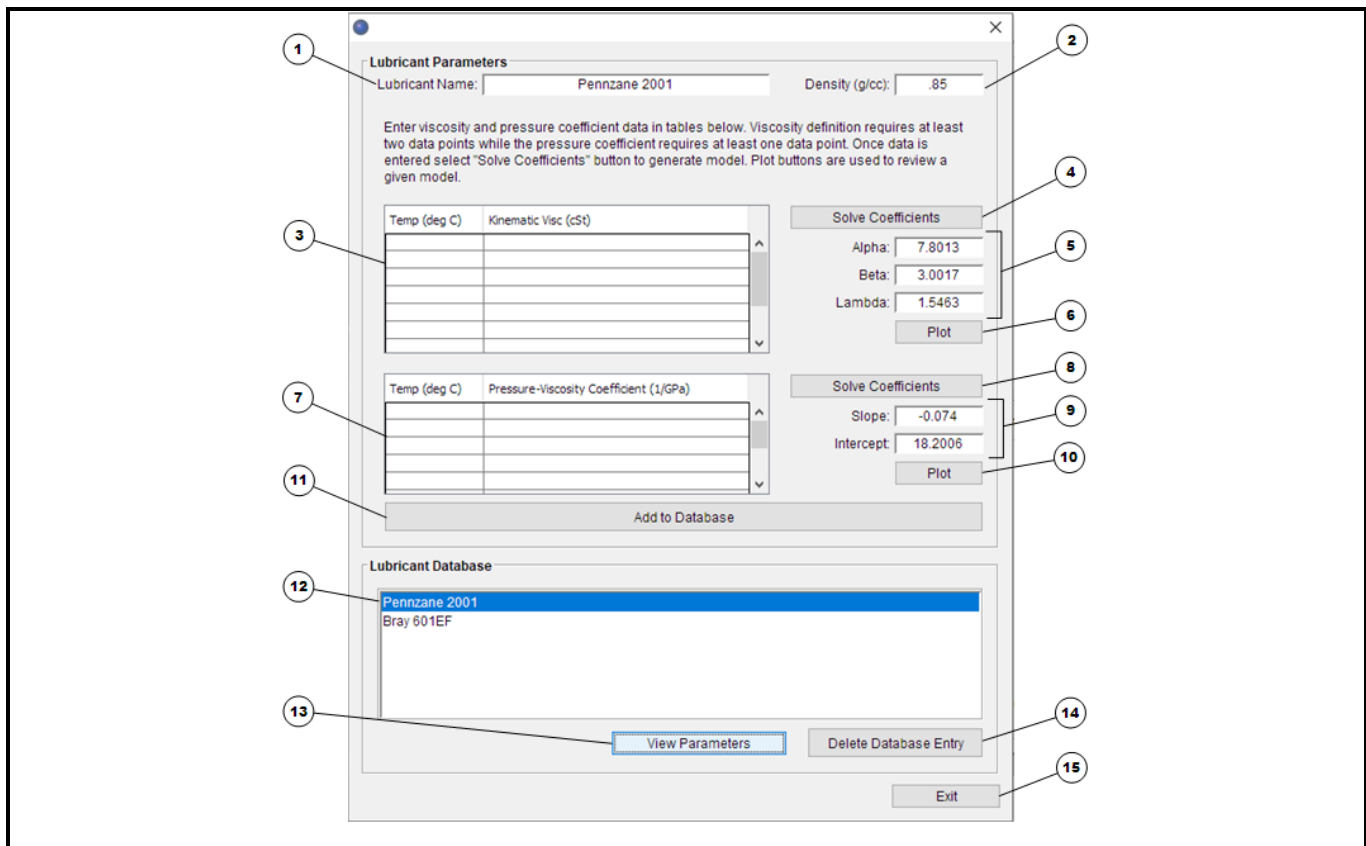


#	Title	Description
1	Parameter Inputs	Each material requires the following parameters: <ul style="list-style-type: none"> • Material Name: Specify a unique name for the material database entry • Young's Modulus (psi): Specify the modulus of elasticity of the material • Poisson's Ratio: Specify Poisson's ratio for the material • Specific Density (lbm/in³): Specify the density of the material • Coefficient of Thermal Expansion (in/in-F): Specify the coefficient of expansion for the material
2	Clear Entries Button	Clears all input entries (does NOT clear the database entries).
3	Add To Database Button	Adds the new entry to the Material Database.
4	Material Database	Shows the all defined entries in the Material Database.
5	View Parameters Button	Displays the parameters of a selected material database entry.
6	Delete From Database Button	Deletes the selected database entry from the database.
7	Close Button	Closes the Bearing Database Editor.

Figure 3.2-8. Material Database Inputs

3.2.3 Lubricant Database Editor

The lubricant database editor allows the user to define, edit or delete their lubricants. ORBIS version 3.0 now model's lubricant properties across temperature. This new capability is not compatible with prior versions and will require the user to re-define their lubricants. See the following figure for a detailed description of the new lubricant editor.



#	Title	Description
1	Lubricant Name	Specify a name for the lubricant.
2	Density (g/cc)	Specifies the density of the lubricant in grams per cubic centimeter.
3	Kinematic Viscosity Vs. Temperature Table	Here the user inputs at least two temperature and corresponding kinematic viscosity data points. It is highly recommended to add more than two points in order to improve the curve fit model.
4	Solve Coefficients Button	After entering kinematic viscosity data you must push this button to generate a curve fit of the viscosity data. A plot window will appear showing the entered data and resulting viscosity model.
5	Viscosity Model Coefficients	These entries are solved for once selecting the 'Solve Coefficients' button. The coefficients match the general model described in ASTM D341 (Annex) with one exception: Lambda will only be 0.7 if there are only two data points specified. Otherwise, Lambda is refined such that the model matches the data in a least squares sense.
6	Viscosity Plot Button	This button allows the user to generate a plot of the viscosity model at any temperature range and step size.
7	Pressure-Viscosity Vs. Temperature Table	Here the user inputs at least one temperature and corresponding pressure-viscosity coefficient. Again, adding more than one data point will improve the model generated.
8	Solve Coefficients Button	After entering P-V coefficient data in table select this button to generate the model coefficients.
9	P-V Model Coefficients	These entries are solved for once the 'Solve Coefficients' button is pressed. P-V model is a simple linear model.
10	P-V Plot Button	This button allows the user to generate a plot of the P-V model at any temperature range and step size.
11	Add to Database Button	Once all inputs are defined and satisfactory model coefficients for both kinematic viscosity and P-V coefficient have been generated this button is selected to add the lubricant to the database.
12	Lubricant Database	This window shows all of the user defined database entries. Entries are selectable by left mouse click.
13	View Parameters Button	Upon selection of a lubricant in the database, selecting this button will populate the Lubricant Parameters into the dialog.
14	Delete Database Entry	Selecting this button with a database entry selected will delete the lubricant from the user database.

Figure 3.2-9. Lubricant Database Inputs

3.3 Analysis Results Window

The Results Window will automatically appear as the top window once an analysis is submitted and ORBIS is able to converge to a solution within the specified convergence criteria. This window provides a comprehensive set of results that can be printed to a final report. Refer to section 5 for a detailed description of all available output.

All results windows ‘float’, which enables the user to keep the results from an analysis run active; then return to the main window and modify their setup and submit an altered analysis. The user can then compare both analysis results side-by-side. Furthermore, the result window is essentially a text pane that allows the user to copy/paste and even type inside.

Since results are categorized there are check boxes that allow the user to show/hide any combination of result sections they want. See the following figure for a detailed description of the Results Window.

Typically an analysis result is not saved within ORBIS. However, options do exist to save the results to a text file or print them. Since ORBIS generally produces analysis results within a fraction of a second the primary means of saving an analysis is to save the analysis setup from the file menu on the main window. The saved file can then be reopened and all original analysis setup parameters will be restored. To produce the results window the user simply selects the Analyze button to recreate the results window.

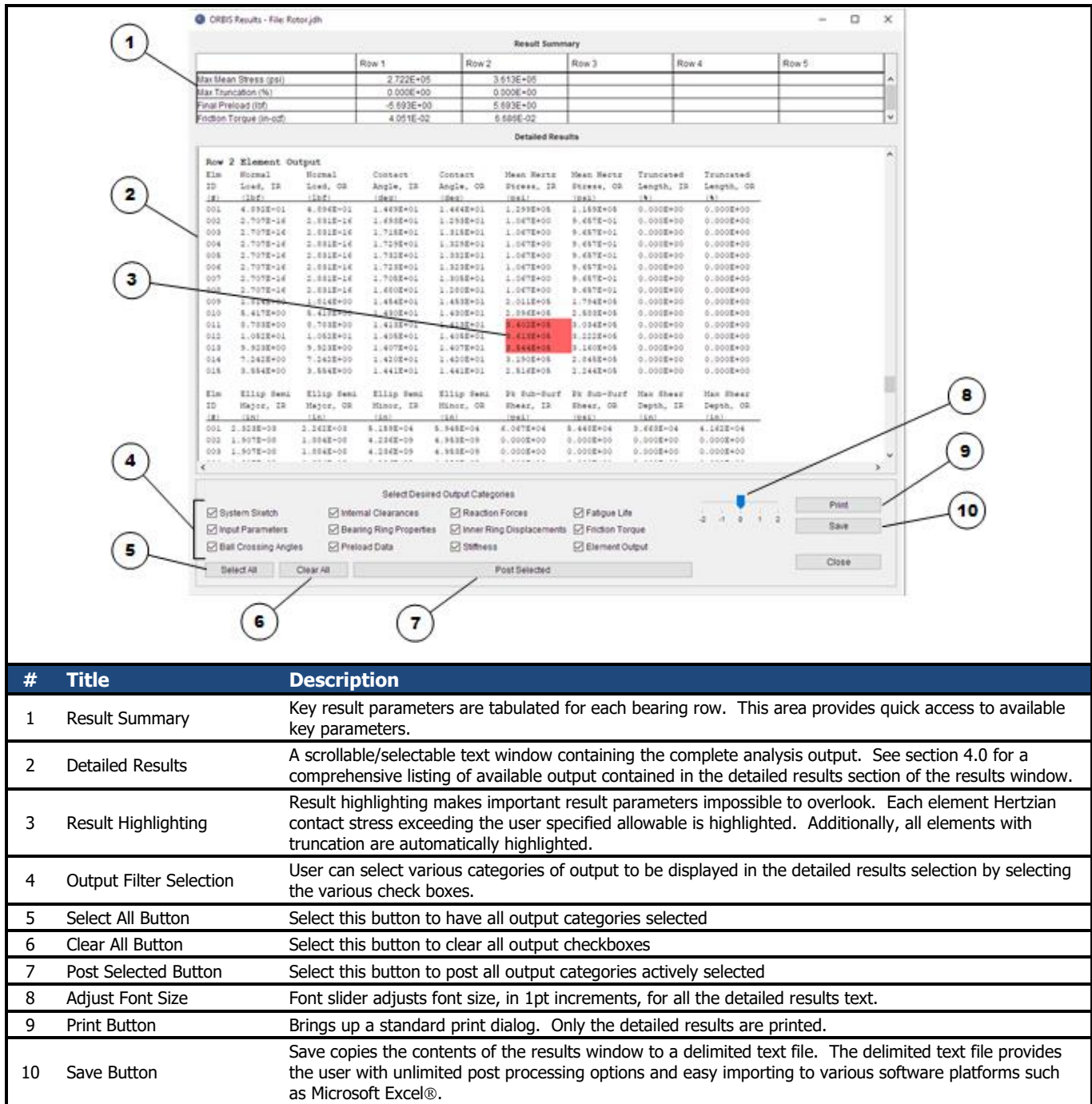


Figure 3.3-1. Results Window

4 Brief Technical Background

ORBIS uses numerical techniques to solve the nonlinear elastic behavior of the user defined system of one or more bearing rows. The model considers each ball-to-race contact for all bearing rows defined in the system; resulting in complete knowledge of the element load distributions and their raceway attitudes. A solution to the system is achieved when the sum of all bearing row reaction forces is sufficiently close to the external applied

forces (system equilibrium). Core analysis methods follow the mathematical theories developed and published by A.B. Jones (1964).

The system model essentially follows the logical process necessary to assemble a rotational system: initial conditions are defined, the bearings are fit into the assembly, preload is applied to the bearings, and external loading is finally applied to the mounted and preloaded bearing system. The parameters describing relative axial ring displacements and internal clearance changes are tracked at each step of the process; ultimately leading to the final state of the bearing system.

Since bearings typically operate with very small internal clearances it is important to consider the effects of boundary conditions on the races when establishing the mounted state of the bearing system. ORBIS implements a compliance model based on classical thick ring theories to determine the final mounted and preloaded state of the bearing system. The compliance model is also used for assessing thermal strains. Once all mounting, preloading and thermal strains have been considered the traditional fixed ring theories of are applied to assess effects due to external loading.

The compliance model makes a key assumption that, in the local vicinity of the bearing, the housing, shaft and bearing rings can be expressed with a series of nested rings. Thick ring theory assumes the representative rings have uniform constant wall thickness and all deflections remain within the linear elastic region of the material. Since the groove side of a bearing race clearly has a non-constant cross section an equivalent diameter has been developed (reference section 5.7 for the equivalence model) to represent bearing rings. The equivalence model is shown in section 5.7 and has been correlated to detailed finite element models of standard bearing ring cross-sectional proportions. The compliance model also handles various sudden changes in the boundary conditions of each nested ring, such as when the fit between a bearing O.D. and housing I.D. transitions from clearance to interference due to preload expansion or thermal expansion.

4.1 Preloading

Axial preloading creates radial forces on the bearing rings that increase the bearing's internal clearance—the rings stretch—and change the inner-to-outer ring stickout (axial offset of ring faces when bearing is in its free contact angle position). To find the preloaded state, with account for ring stretch, each bearing row is iteratively solved until the change in internal clearance equals the resultant ring deflection due to preload pressure on the raceways. This algorithm also accounts for nonlinear behavior of axial load versus deflection and potential nonlinear radial ring stiffness due to fitup gap closure.

The preload value entered within each bearings row's preload input (see section 3.1.4) includes options to specify the preload *condition* and preload *type*. These parameters, as discussed below, allow the user to specify at what state the entered preload value is true.

4.1.1 Preload Condition

Since ORBIS treats bearing rings as flexible objects, knowledge of an initial state is needed to determine the final solution. The preload condition radio buttons (reference section 3.1.4) are used specify whether the preload value, as input by the user, applies at the mounted or un-mounted condition.

The term 'mounted' means the bearing has been fit, or mounted, into the shaft/housing system and all boundary influences from interference fitting and ring clamping are included. This means the specified preload will be exactly as input at the mounted state of the bearing. The compliance model will determine what the resulting mounted and preloaded contact angle is and use that as the starting point for any external loading.

Conversely, an un-mounted preload condition means the specified preload value is true when the bearing rings are pressed together axially but are radially unrestrained (free to expand/contract). This is very common for pre-ground matched duplex pairs supplied by most bearing manufacturers. For an un-mounted preload condition, it is not necessarily true that the final mounted preload will equal the preload input by the user. This difference will depend on the influence of mounted boundary conditions, such as interference fitting, and the specified ***Preload Type***.

4.1.2 Preload Type

ORBIS handles two different preload types: rigid and spring (reference section 3.1.4). The preload type determines how the preload reacts to mounting and thermal influences.

When the preload type is specified as ‘rigid’ there is no further axial motion allowed between the rings and their adjacent boundary (shaft or housing). Additionally, when the preload condition is specified as un-mounted, the preload gap determined from radially unrestrained rings will be enforced during the mounting routine. If the mounting fits are tighter than the ring expansions due radially unrestrained preloading then this preload gap enforcement will always result in a mounted preload that is higher than the specified un-mounted value. Perhaps the best way to describe a rigid preload type is when one uses a thick clamp plate, or jam nut, that is torqued down on the bearing with forces much higher than the bearing preload to ensure all pre-ground face gaps have been securely seated.

When the preload type is specified to be ‘Spring’ then relative motion between the inner or outer rings and their mating shaft or housing is permissible. Once the preload force is overcome the bearing reaction follows the spring rate curve. Spring type preloading essentially bypasses unmounted preload specifications since typical spring rates prohibit noticeable force increase for small deflections—this assumption will be valid for spring rates much lower than the axial stiffness of the bearing, which is often the case.

4.2 Convergence Criteria

ORBIS uses the IEEE 754 technical standards for all floating-point arithmetic. All calculations use at least 64-bit precision. Key calculations pertaining to matrix inversion and the overall system Jacobian are extended to 128-bit precision to improve accuracy of the solver. The default criteria for convergence are show below. Refer to section 3.1.6.7 for instructions on how to change these settings.

Table 4.2-1. Solver Convergence Criteria

Parameter	Default Error (\pm)	Comments
System Equilibrium Error	0.001%	The allowable system equilibrium error is defined as a percentage of the applied external loading (Euclidean norm of force components). Error is defined as the difference between the norm of all bearing row reaction forces and the norm of the external applied forces. In the case where there is zero external loading the error defaults to 1.0E-5 lbf.
Preload Force Error	1.0E-5 lbf.	Preload force error is defined as the difference between the bearing reaction forces and the applied preload force.
Ring Expansion Error	1.0E-7 in.	The ring expansion error is defined as the difference between ΔP_D input into the Jones model (a 'fixed ring' model) and the resulting ΔP_D due to ring deflections. Ring deflections are determined by using the resulting ball normal forces and associated contact angle to determine an equivalent radial pressure on the bearing ring.

5 Output Descriptions

The following sections describe the analysis output generated by ORBIS.

5.1 Results Summary

The result summary table from the Results Window is copied to the detailed results section so that it may be included in the printable report.

5.2 Input Parameters

This section provides a list of the user inputs used to generate the results. The section is divided into system inputs, bearing inputs, and row inputs. System inputs include shaft/housing material names, system rotational speeds, lubricant name and lubricant properties for each row. Bearing inputs contain all geometry used to define each bearing within the user's setup. Row inputs include all row information for each row (i.e. row locations, fitups, preloads, clamp loads, spring rates, etc).

5.3 Material Properties

The material properties section provides a list of the materials used in the system along with the defined properties. Properties include Young's Modulus, Poisson's Ratio, specific density and Coefficient of thermal expansion.

5.4 External Applied Loads

This section displays the user defined system load components and the application point at which they are applied (location is along the x-axis). Loading information for up to three separate load points will be displayed.

5.5 Ball Crossing Angles

Ball Crossing Angles are displayed for each row and are defined as the angular rotation required by a given ring, inner or outer, to cause a ball to travel to an adjacent ball station. These results are based on the mounted bearing contact angle. These angles could be used with the rotational speed to provide characteristic disturbance frequencies for inner or outer raceway defects.

5.6 Internal Clearances

This section provides bearing internal clearances in the diametral (denoted by ‘PD’) and axial (denoted by ‘PE’) directions. Note: diametral clearance is also commonly referred to as ‘radial play.’ Diametral play is defined as the total linear radial distance the inner ring can move relative to the outer ring with negligible applied force. Axial play is defined as the total axial displacement the inner ring can move relative to the outer ring.

Result descriptions for each of the internal clearance outputs are as shown in the following table.

Table 5.6-1. Internal Clearance Output Descriptions

Result Title	Parameter Description
Diametral, PD (free)	Radial play of the bearing in its unmounted, or free, state.
Axial, PE (free)	Axial play of the bearing in its unmounted, or free, state.
Delta PD Mount	Change to radial play due to interference and clamping effects (ambient temperature)
Delta PD Preload	Change to radial play due to preloading (ambient temperature)
Delta PD Temp (free)	Change to radial play due to thermal expansions of bearing rings and balls. This is a free bearing condition with applied temperatures.
Delta PD Temp Mount	Change to radial play due to interference and clamping effects at temperature. Fitup at bearing ID and OD are adjusted based on relative thermal expansions. Ring clamping forces are always as input by user and do not get adjusted for temperature.
Delta PD Temp Prld	Change to radial play due to effective preload at temperature. This change represents the difference in radial play from the mounted temperature state to the mounted and preloaded state.
Diametral, PD (tot)	Final effective radial play after all mounting, preloading and temperature effects.
Axial, PE (tot)	Final effective axial play after all mounting, preloading and temperature effects.

5.7 Bearing Ring Properties

This section provides output for the equivalent raceway diameters and the bearing inner and outer diameter expansions due to mounting, preloading and temperate effects. The equivalent raceway diameter is an approximated constant wall representation of the groove side of the bearing ring and is used for the mounting/preloading algorithm (thick-ring theory). The equations for equivalent diameters are defined as function of the bearing geometry as shown in the following figure.

$$D_{IR} = d_{IR_{race}} + 0.68(h_l + h_d) - \frac{2h_l}{d_{IR_{race}}}(h_l - h_d)$$

$$D_{OR} = d_{OR_{race}} - 0.68(h_l + h_d) + \frac{2h_l}{d_{OR_{race}}}(h_l - h_d)$$

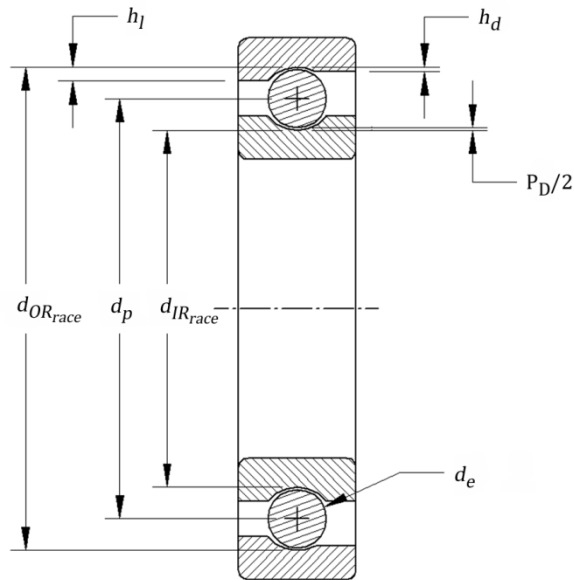


Figure 5.7-1. Bearing Nomenclature for Equivalent Ring Calculations

The following table provides a detailed description of each of the different bearing ring outputs.

Table 5.7-1. Bearing Ring Output Descriptions

Result Title	Parameter Description
I.R. Equiv Diam (in)	Equivalent inner ring diameter as shown in FIG 23 equations
O.R. Equiv Diam (in)	Equivalent outer ring diameter as shown in FIG 23 equations
I.D. Expand, Free (in)	Expansion of bearing inner diameter due to preloading the bearing in a free, radially unrestrained, condition. Applies at ambient temperature.
I.D. Expand, Mnt (in)	Expansion of bearing inner diameter due to ambient mounted and preloaded state. This includes effects from interference fits, ring clamping and preloading.
I.D. Expand, Temp (in)	Expansion of bearing inner diameter due to mounted and preloaded state at temperature. This includes fitup changes due to temperature, ring clamping, and effective preload at temperature.
O.D. Expand, Free (in)	Expansion of bearing outer diameter due to preloading the bearing in a free, radially unrestrained, condition. Applies at ambient temperature.
O.D. Expand, Mnt (in)	Expansion of bearing outer diameter due to ambient mounted and preloaded state. This includes effects from interference fits, ring clamping and preloading.
O.D. Expand, Temp (in)	Expansion of bearing outer diameter due to mounted and preloaded state at temperature. This includes fitup changes due to temperature, ring clamping, and effective preload at temperature.
I.D. Fitup, Mnt (in)	Effective fitup (diametral interference/clearance) between bearing inner diameter and shaft outer diameter at mounted and preloaded state at ambient temperature. This includes fitup changes due to interference, ring clamping, and preloading.
I.D. Fitup, Temp (in)	Effective fitup (diametral interference/clearance) between bearing inner diameter and shaft outer diameter at mounted and preloaded state at temperature. This includes fitup changes due to interference, ring clamping, and preloading.
O.D. Fitup, Mnt (in)	Effective fitup (diametral interference/clearance) between bearing outer diameter and housing inner diameter at mounted and preloaded state at ambient temperature. This includes fitup changes due to interference, ring clamping, and preloading.
O.D. Fitup, Temp (in)	Effective fitup (diametral interference/clearance) between bearing outer diameter and housing inner diameter at mounted and preloaded state at temperature. This includes fitup changes due to interference, ring clamping, and preloading.

5.8 Preload Data

Preload data output shows the applied preload, which pertains to whatever preload condition was specified, the mounted preload, the mounted preload at temperature and some corresponding ring deflections. Refer to the following table for a description of each output.

Table 5.8-1. Preload Output Descriptions

Result Title	Parameter Description
Applied Preload (lbf)	The applied preload is the value entered into the preload input field. This means it's applicability depends on the preload condition.
Mounted Preload (lbf)	Mounted preload pertains to the preload after all mounting conditions have been applied. For example, if your system of bearings has been installed with all clamp loads and fitups, and is at ambient temperature with no external loading, this would be the mounted preload.
Dx, Mounted Prld (in)	Shows the relative motion of the inner ring with respect to the outer ring in the mounted preload state. Zero is defined to be the initial contact of the bearing at its free contact angle with no external influences.
Preload at Temp (lbf)	Preload at temperature is the mounted preload with temperature expansions applied to the system. No external loading applied.
Dx, Prld at Temp (in)	Shows the relative motion of the inner ring with respect to the outer ring in the 'Preload at Temp' state.

5.9 Reaction Forces on Shaft

This section provides the resulting bearing reaction force components, expressed in global coordinates, on the shaft at each bearing row. These forces include all mounting, preloading and external loading conditions and apply at the center of their respective bearing row locations.

5.10 Inner Ring Displacements

This section provides the total displacement, expressed in global coordinate components, of the inner rings due to mounting, preloading, temperature and external loading. Initial, or zero position, is therefore the unmounted free condition. Note: the axial (dx) displacements due to external loading only can be obtained by taking the difference between the total displacements shown and the axial displacements due to preloading as shown in the Preload Data output section (section 5.8).

5.11 Stiffness Outputs

ORBIS provides three different types of stiffness outputs: axial stiffness with ring compliance considerations, system Jacobian diagonal terms, and complete 5x5 stiffness derivatives for each bearing row. Refer to subsequent sections for a description of each type of stiffness result.

5.11.1 Axial Stiffness with Ring Compliance

This result provides the system axial stiffness, at the mounted and preloaded state, with compliant ring considerations. This output includes the contribution of ring expansion due to an incremental axial load with the traditional bearing contact stiffness to more accurately represent axial stiffness of the mounted and preloaded system. No influences from external loading are included in this stiffness output.

5.11.2 System Jacobian

This result provides the system Jacobian diagonal terms. Results apply in the fully loaded condition. Additionally, the system Jacobian assumes a rigid system during external load application. Results are provided at the system center and each specified load point.

5.11.3 Stiffness at Load Point

When performing a flexible shaft analysis, the output contains additional complete stiffness matrices for each load point specified in the system. This matrix represents the full 5 x 5 stiffness matrix of the system at the specified load point. Stiffness results include effects of shaft compliance and bearing stiffness's in their final loaded equilibrium state.

5.11.4 Row Stiffness Matrix

This section provides complete 5x5 stiffness derivatives for each bearing row. Stiffness results correspond to the quasi-static equilibrium state of the system in its final loaded state.

5.12 Fatigue Life

This section provides individual ring and total system fatigue cycles for various conditions. Note: Fatigue results are only generated for dynamic analysis runs. Outputs include L10 life, adjusted L10 life, and adjusted life with consideration for film thickness. L10 Fatigue life calculations are based on the Lundberg – Palmgren theories as shown in Jones (1964). The adjusted life output is based on the life factor theory adopted in AFBMA (1990) standards. This result includes the user defined life factor input along with a computed factor based on the user reliability input.

The life factor input allows the user to enter a combined factor to account for items such as materials, cleanliness, and misalignments. Bamberger (1971) provides a useful reference for computing various life factors. The adjusted life with film includes an additional lubricant factor, which is also provided for reference. The film parameter is from Bamberger (1971) and follows the AFMBA recommended average curve and uses the minimum film parameter as discussed below. All fatigue calculations use the individual rolling element results and do not require determination of an equivalent radial load on the bearing row.

5.13 Bearing Torque

Bearing torque results are only generated for dynamic analysis runs. Two types of torque are considered: friction torque and viscous torque. Friction torque output represents the torque associated with rolling and spinning within the contact area when the balls start rotating. Precisely, this is the torque due to interfacial slip (aka Heathcoate Slip) at the contact ellipse. ORBIS uses the 'Race Control' theory from Jones (1964) for 2PT bearing types and therefore only allows spin to occur on one raceway. 4PT bearing types use a minimum energy theory that allows spin and slip to occur simultaneously. The computed torque output can be scaled with the coefficient of friction input parameter. Reference Jones (1964) for the friction torque calculations used by ORBIS.

Viscous torque output is based on the Palmgren model, which was republished by Harris (2001). This model accounts for lubricant viscosity and requires use of a viscous torque factor. Hence, the viscous torque output can be scaled by direct modification of the viscous torque factor.

To achieve optimal torque predictions the user will need to tune both the coefficient of friction at the ball contact and the lubricant viscous torque factor. This is most accurately done by use of existing test data of a known configuration. The coefficient of friction should be tuned first based on breakaway or slow speed test data. The viscous torque factor is then tuned based on test data for two different operational speeds. Note that viscosity is highly sensitive to temperature.

5.14 Ball Excursions

Dynamic runs include output showing the maximum ball excursion for each bearing row. This output represents the maximum circumferential excursion, in units of inches, a ball travels relative to the average ball path. Another way to view this is the maximum amplitude a ball travels within the retainer pocket. This result is achieved by taking the orbital velocities of each ball, determining the average velocity of all balls, integrating the orbital velocities through one full revolution, and finally computing the difference between actual integrated ball positions and the average ball positions. Final reported excursion represents the maximum ball departure from the average.

5.15 Row Outputs (Element-Wise results)

Detailed outputs for each element of each row are provided in tabular form. Output tables are repeated for each bearing row in the user's system. These tables differ depending on whether the analysis is static or dynamic. All outputs correspond to the system equilibrium state after application of all external loading.

Results for a 4PT bearing will have detailed element output for all four possible contacts. The different contacts are distinguished with the following terminology: IR-L would read 'inner ring left' and thus denotes the left side of the inner race, OR-R would be the right side of the outer raceway.

5.15.1 Element Number

The Element Number is simply an indexing scheme to identify each of the rolling-elements uniquely. The first ball is always placed along the $-Z$ axis (this is the three o'clock position when looking down the $-X$ axis at the $Y-Z$ plane) and increments in an increasing fashion toward the $+Y$ axis.

5.15.2 Normal Ball Load

The Normal Ball Load is the load applied by each ball into each raceway contact. This load is directly normal to the contact ellipse and has an attitude defined by the contact angle.

5.15.3 Contact Angle

The Contact Angle output describes the angle of the normal ball load vector to the plane extending through the $Y-Z$ plane.

5.15.4 Mean Hertz Stress

The Mean Hertz Stress output represents the average Hertzian contact stress over the elliptical contact area. Peak stress for an elliptical contact can be computed by multiplying the mean stress by $3/2$. ORBIS will automatically highlight, in red, all values that exceed the user defined allowable mean Hertzian stress from the main user interface. All stress results assume the contact ellipse is fully contained within the raceway.

5.15.5 Truncation Analysis

When truncated elements are found additional output is provided directly after the first table of the affected row. This output computes peak center stresses and peak edge stresses for all elements exhibiting truncation. The method used follows the publication by Frantz and Leveille (2001). This output reports peak, not mean, stresses and all edge stresses include a nominal 1.8X edge concentration factor. Since this factor is applied directly to the stress, the user may manually modify edge stresses based on alternate edges stress factors as they see fit.

5.15.6 Truncated Length

The Truncated Length represents the percent of the total length of the contact ellipse, along the major axis, that is truncated due to shoulder or dam override. ORBIS will automatically highlight all elements that have any truncation.

5.15.7 Ellipse Semi Major

The Ellipse Semi Major output represents one half of the major dimension of the contact ellipse.

5.15.8 Ellipse Semi Minor

The Ellipse Semi Minor output represents one half of the minor dimension of the contact ellipse.

5.15.9 Max Sub-Surface Shear

The Max Sub-Surface Shear is the peak shear stress developed below the raceway surface due to contact stress.

5.15.10 Max Shear Depth

The Max Shear Depth is the distance along the normal to the contact area, below the raceway surface, at which maximum shear stress is developed.

5.15.11 Upper Edge Location

The Upper Edge Location represents the edge of the contact ellipse that is closest to the land diameter. This value is represented as a ratio of its height from the center of the raceway to the ball diameter.

5.15.12 Lower Edge Location

The Lower Edge Location represents the edge of the contact ellipse that is closest to the dam diameter. This value is represented as a ratio of its height from the center of the raceway to the ball diameter.

5.15.13 Contact Normal Approach

Contact Normal Approach represents the total combined deflection of the contacting bodies (rolling-element and raceway). This deflection is along the normal direction to the contact area.

5.15.14 Contact Normal Stiffness

Contact Normal Stiffness represents the stiffness of the rolling-element to raceway contact area stiffness in the normal direction.

5.15.15 Spinning Velocity

Spinning Velocity is the angular velocity of the rolling element about the axis of rotation that is normal to the contact on the un-controlling race. Per Jones' (1964) race control theory, spin can only occur on one raceway while pure rolling occurs on the other. Based on the spinning velocity output one can deduce race control (i.e. if there is zero spinning velocity on a given raceway than that raceway is 'in control').

5.15.16 Rolling Velocity

Rolling Velocity is the relative angular velocity of the rolling element about its own axis of rotation parallel to the contact on the controlling race.

5.15.17 Spinning Torque

Spinning Torque is the component of torque generated by interfacial slip within the contact area due to rolling-element spin.

5.15.18 Rolling Torque

Rolling Torque is the component of torque generated by interfacial slip within the contact area due to pure rolling.

5.15.19 Element Roll Velocity

The Element Roll Velocity represents the rotational velocity of the rolling elements as seen relative to the pitch orbit velocity.

5.15.20 Pitch Orbit Velocity

The Pitch Orbit Velocity is the rotational velocity of the bearing pitch diameter about its spin axis. This is essentially the angular velocity of rolling-element cage or retainer.

5.15.21 Minimum Film Height

The Minimum Film Height represents the thinnest point of the lubricant along the center line of the contact ellipse. Two types of minimum film heights are provided: H-D, which stands for Hamrock-Dowson, and C-Z, which stands for Coy-Zaretsky. The H-D calculation is based on the 'Hard-EHL' theory by Hamrock and Dowson (1981) for fully flooded conditions. The C-Z calculations apply a starvation correction factor to the H-D results.

5.15.22 Minimum Lambda Value

Lambda is a dimensionless parameter that is often used to describe the lubricant regime of the bearing. Its value is determined by taking the ratio of the minimum film height to the root sum squares (RSS) of the contacting surface roughness. Mathematically, lambda is defined as follows. The lambda values output in ORBIS are based on the C-Z minimum film height.

$$\lambda = \frac{h_{min}}{\sqrt{R_{raceway}^2 + R_{ball}^2}}$$

5.15.23 Centrifugal Force

The Centrifugal Force output represents the radial body force of the rolling element due to its orbital velocity and mass. This force tends to create differing contact angles between the inner and outer race contacts and is treated in the analysis per Jones (1964).

5.15.24 Gyroscopic Moment

The Gyroscopic Moment output represents the spinning body moment of the rolling element due to its angular velocity and inertia. The influences of this force are treated in the analysis per Jones (1964).

5.15.25 Ball Spin Axis Angle

The ball spin axis angle defines the instantaneous axis of spin for each ball. This result is only available for 4PT bearings. Further definition of this angle can be found in Jones (1964) and its derivation for 4PT bearings is described in Halpin (2016).

6 References

- Anon.,(1990) *Load Ratings and Fatigue Life for Ball Bearings*, ANSI/AFBMA 9-1990, The Anti-Friction Bearing Manufacturers Associations, Washington, DC.
- Bamberger, E. N. (1971) *Life Adjustment Factors for Ball and Roller Bearings – An Engineering Design Guide*, American Society for Mechanical Engineers, New York
- Frantz, P. P. and Leveille, A. R. (2001), *An Approach to Predicting the Threshold of Damage to an Angular Contact Bearing During Truncation*, Aerospace Report No. TR-2001(8565)-4, El Segundo, CA.
- Halpin, J. D. and Tran, A. N. (2016) *An Analytical Model of Four-Point Contact Rolling Element Ball Bearings*, ASME J. Tribology, 2016, doi:10.1115/1.4033134
- Hamrock, B. J. and Dowson, D. (1981) *Ball Bearing Lubrication – The Elastohydrodynamics of Elliptical Contacts*, Wiley, New York, N.Y.
- Harris, T. A. (2001), *Rolling Bearing Analysis*, 4th Edition, John Wiley & Sons, New York, NY
- Jones, A. B. (1964) *The Mathematical Theory of Rolling-Element Bearings*, in *Mechanical Design and Systems Handbook*, H. A. Rothbart, ed., McGraw-Hill, New York, N.Y. article 13.
- Todd, M.J. and Johnson, K. L. (1986) *A Model for Coulomb Torque Hysteresis in Ball Bearings*, in *International Journal Mechanical Science*, Vol 29, No. 5, pp 339-354, Great Britain.